James Arthur

Dr. James Arthur received his Ph.D. from Yale University in 1970 and taught at Yale from 1970 until 1976. After that, he joined the faculty of Duke University in 1976. He has been a professor at the University of Toronto since 1978. As a student of Robert Langlands, Arthur works on number theory and his major research interest includes representations of Lie groups and automorphic forms.

Arthur was elected a Fellow of the Royal Society of Canada in 1981 and a Fellow of the Royal Society in 1992. He was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 2003 and a member of the National Academy of Sciences in 2014. He was awarded the Wolf Prize in Mathematics in 2015 and received the Leroy P. Steele Prize for lifetime Achievement by the American Mathematical Society in 2017.

Memories of a Conversation with Yau from Long Ago

It was during the winter term at the Institute for Advanced Study in 1984. Yau was still a member of the Mathematics Faculty, while I was a visiting member in the special program that year in automorphic forms and the trace formula. Yau sat down opposite me at what was known informally as the "mathematics table". In point of fact, most of the mathematicians had already vacated the table, having finished their lunch, and were back at their desks or seminar rooms. We had the table almost to ourselves.

Yau began by asking me about the spectrum of arithmetic, locally symmetric Riemannian manifolds. These were the objects (in differential geometric guise) that were being studied in the special program. The most fundamental of them are noncompact (even though they have finite volume with respect to the Riemannian measure). I say "asked", but I quickly realized that Yau already knew a great deal about them. He was coming to them through his extensive background and deep studies of general Riemannian manifolds. He then told me something that I should have known, or perhaps did know but had forgotten. Namely, that generic, noncompact Riemannian manifolds were expected to have no discrete spectrum. We were of course speaking of the spectrum with respect to the natural Laplace-Beltrami operator. No discrete spectrum means that there are no square integrable eigenfunctions of this operator on the manifold.

In his typical fashion, Yau put the matter in clear and dramatic terms. He knew that as noncompact Riemannian manifolds, the arithmetic spaces had a rich spectrum. Indeed, it is their eigenvalues, together with the supplementary eigenvalues attached to other operators (p-adic Hecke operators) that commute with the Laplacian, which are at the heart of the Langlands program. They are conjectured to govern much of what happens in the arithmetic world. But Yau also knew what he had pointed out to me, that typical noncompact Riemannian manifolds will have no discrete spectrum. The contrast is indeed striking.

We spent the rest of the lunch with a very agreeable discussion of the discrete spectra of arithmetic spaces, and the possible implications, both mathematical and philosophical, for their absence from more general Riemannian spaces. The entire conversation left me with indelible memories. I was already well motivated to continue working on arithmetic spectra. But to understand that they were the truly singular objects within the universe of all noncompact Riemannian manifolds was quite overwhelming. It gave me further perspective on how extraordinary mathematics can be. I suspect this is precisely what Yau had intended.