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Aerial Film Granularity and Its Influence on Visual Performance

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Abstract

Extensive measurements of granularity have been carried out on aerial film emulsions developed under different processing conditions at a number of densities using a variety of aperture configurations. Spectral analyses of this data have been performed to determine Wiener or noise power spectra of granularity, and the theoretical information capacity of aerial film emulsions. Representative relationships between measuring parameters and measured properties have been determined.

Proposals have been made on the appropriate parameters of granularity to express its effect on the performance of the visual observations of pointing and interpretation in photogrammetry.

1. Introduction

The ability of observers to measure and interpret photographic details is a function of many physical and psychophysical factors. A substantial volume of research has been carried out to isolate the effects of a number of these factors. Visual performance can therefore be predicted more closely, and in addition, conditions of observation can be designed so that optimum performance can be achieved.

In photogrammetry, the components of the imaging system which are associated with the recording of the photographic image, together with those associated with image reproduction and observation all affect the quality of the image viewed by the observer, and thus his ability to perform specific visual tasks. Attempts have been made to determine one or more parameters to describe the characteristics of imaging systems in a global fashion, and thereby relate visual performance to such factors, with varying degrees of success. The purpose of this paper is to describe phase one of a research project to measure granularity and modulation transfer functions (MTF) of aerial photographic films commonly used in practice. Such data will act as information to be incorporated in image assessment models to be studied in phase two of the project. The effects of granularity to-date have been largely neglected in such image assessment models. Improved knowledge of all factors is important as attempts are being made to achieve higher precisions of measurement to photogrammetric points and improvements in the interpretability of details on aerial photography.

2. Granularity

2.1 Description and Measurement

Granularity is the objective measure of inhomogeneities in a photographic image, brought about by the granular characteristics of photographic emulsions. By contrast the term graininess is the subjective sensation by an observer of granularity. An image assessment model must relate these two measures if the effects of the physical characteristics of an amulsion on visual performance are to be properly considered.

The locations of the silver halide grains within the emulsion are random, and therefore granularity is a statistical quantity, subject to variations depending on the local position of the density measurement. A commonly used measure of granularity is the Root Mean Square (RMS) variation in density derived from a large number of density measurements across the film sample. For a film of specific density, developed under specific chemical conditions, the granularity may be assumed to be constant.

$$\mathrm{RMS} = \sigma_D = \sqrt{\sum_{i=1}^{n}(D_i - \overline{D}^2/n} \qquad (1)$$

where D_i is the local density value at position i

$\overline{D}$ is the mean density of the film over n points.

Granularity is measured in a microdensitometer with an aperture of specific size and shape, normally rectancular or circular, the RMS values varying accordingly. The standard Kodak measure of granularity, for example, is derived using a 48µm diameter aperture, the resulting value being multiplied by 1000. Granularity of the Kodak 2405 film (Double-X) at a density of 1.0 is therefore quoted as 36. (Kodak 1970).

The well-known Selwyn relationship between aperture and RMS for black and white flashed uniform densities states that for circular apertures the product of σ_D and the diameter, d, of the scanning aperture should be a constant (Selwyn, 1935). This relationship has been proved to be inexact in some studies but generally research has confirmed its existence (e.g. Stulz et al, 1959). The variation of σ_D with aperture size, density and film gamma will be investigated in this paper for aerial film emulsions.

More recent studies have incorporated Fourier techniques to derive the noise power or Wiener spectrum of granularity. This involves determination of the correlogram by self-convolution of the density trace, and the subsequent Fourier transformation as described in the following section.

2.2 Correlogram and Wiener Spectrum

The correlogram derived from a one-dimensional scan is (Zweig, 1956a):-

$$\phi(t) = \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} f(x)\ f(x + t)dx \tag{2}$$

where f(x) is the measure density trace

t is the delay term associated with the self-convolution computation. Practically, equation (2) computes the mean value of the correlation between the density measured at a position x on the trace, with that measured at position (x + t).

It may therefore be written as (Zweig, 1956b):-

$$\phi(t) = \frac{1}{N} [D(x_i) - \overline{D}][D(x_i + t) - \overline{D}] \tag{3}$$

where $D(x_i)$ and $D(x_i + t)$ are densities measured at locations x_i and $(x_i + t)$ along the trace for N determinations.

$\overline{D}$ is the mean density reading of the whole trace.

Significantly $\phi(0) = \sigma_D^2$, the square of the RMS value. Upon Fourier transformation of $\Phi(t)$ the power or Wiener spectrum $\Phi(u)$ is:-

$$\Phi(u) = \int_{-\infty}^{\infty} \phi(t)\ \exp(-2\pi iut)dt \tag{4}$$

where u refers to spatial frequencies; alternatively

$$\Phi(u) = |F(u)|^2 \tag{5}$$

where F(u) is the Fourier transform of f(x).

As $\phi(t)$ and $\Phi(u)$ are transform pairs, $\int_{-\infty}^{\infty} \phi(t)dt = \Phi(0)$ and the reverse also holds. That is, the area under the correlogram curve, expressed in units of µm, is equal to the central ordinate of $\Phi(u)$.

The correlogram expressed in terms of the two dimensions x,y on the film sample is:-

$$\phi(t,r) = \frac{1}{A} \int f(x)\ f(x + t) \int f(y + r) f(y)dy\ dx \tag{6}$$

where t,r are respectively delay terms applied to x and y coordinates

and A is the region of summation

Therefore, $$\Phi(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(t,r)\exp[-2\pi i(ut + vr)^2]drdt \tag{7}$$

Emulsion granularity for practical purposes may be assumed to be isotropic. That is, the granularity is independent of orientation and location of the x or y axes and only dependent on the vector displacement $z = (x^2 + y^2)^{\frac{1}{2}}$. $\phi(z)$ represents the correlogram and the Wiener spectrum is formulated as follows:-

$$\Phi(w) = 2\pi \int_{-\infty}^{\infty} J_o(2\pi wz)\phi(z)dz \qquad (8)$$

where $w = (u^2 + v^2)^{\frac{1}{2}}$

and J_o is the Bessel Function of order zero.

Equation 8 has once again been reduced to a single variable problem, but the formulation is complicated by the existence of a Bessel Function. Computation by a fast Fourier transform package however is normally possible. As with the one-dimensional case, similar statements regarding the equality of the definite integral of $\phi(z)$ and the central ordinate of $\Phi(w)$ may be made; appropriate dimensions are $(\mu m)^2$.

2.3 Choice of Aperture

The Wiener spectrum should be measured using an infinitely long slit for accurate determinations. (De Belder et al, 1967). However, since it can be shown that slits longer then 100µm are adequate for the determination of Wiener spectrum for spatial frequencies less than 100 lines/mm, a slit of 1µm x 100µm was chosen for this study.

Circular apertures are commonly used for granularity determinations yielding values which are dissimilar to those derived by using a slit. If required, corrections to the observed values are possible (Trabka, 1965), but such determinations are made in order to match the scanning achieved by an observer's visual system, which is clearly dependent on the circular pupil of the eye. It has been shown by Stulz et al (1959) that varying the circular aperture size is equivalent to changing the optical magnification of an observation instrument. For this study the range of circular apertures used is as shown in Table 1. Practical limitations restricted the range of apertures to 4 times. Tests 3 and 4 were both included to analyse effects of different experimental conditions.

TABLE 1

Test No.	Effective Aperture Size	Numerical Aperture of Microdensitometer	Optical Magnification
1	1µm x 100µm Rectangular	0.4	44
2	12.5µm diam. Circular	0.4	44
3	25µm diam. Circular	0.4	44
4	25µm diam. Circular	0.28	22
5	50µm diam. Circular	0.28	22

2.4 Correction for Microdensitometer Optics

The degrading effects of the MTF of the microdensitometer optics will result in a measured Wiener spectrum of granularity (Shaw, 1962)

$$\Phi'(w) = |M|^2 \, \Phi(w) \qquad (9)$$

where M is the MTF of the optics

and $\Phi(w)$ is the corrected Wiener spectrum.

$\Phi(w)$ can therefore be determined provided M is known. The MTF of the optics was determined for this study by scanning across a sharp edge. The slope along this trace as a function of position gives the line spread function while the MTF is the Fourier transform of the line spread function, (Welch 1971). As the MTF's of emulsions (Figure 10) were also required a sharp edge was imaged onto film samples, and the MTF's derived accordingly, subject to corrections for microdensitometer optics. Corrections for effects of microdensitometer optics were not applied to granularity measurements obtained with circular apertures as the effects were negligible for the range of spatial frequencies involved.

An analysis of the microdensitometer measurements using the 2 optical systems for 44x and 22x magnification in Tests 3 and 4, and in subsequent tests, revealed no systematic differences in density readings from the two conditions. It was therefore concluded that the measurement of density in the Joyce Lobel microdensitometer were not affected by incomplete diffusion of the illumination system and the so-called "Callier factor" was therefore assumed to be unity (Schmitt et al 1970).

3. Image Assessment Based on Wiener Spectrum and Information Theory

Attempts to study the image quality of photography incorporating granularity based on Shannon's Information Theory, date back to the 1950's. Fellgett and Linfoot (1955) in particular arrived at a figure of merit for image quality of a photographic system. The method has since been further developed by Linfoot (1959) and others, but while MTF's are regularly used for assessment of photogrammetric systems, the incorporation of Shannon's Information Theory has not been pursued. A major reason for this is that such techniques do not include the observer's capabilities as part of the overall assessment. The method is therefore a purely objective measure of image quality, and additional studies must be undertaken to relate such a measure to observer performance.

Image assessment methods based on Information Theory assume that the photographic film is a noisy communications channel which is limited in its capacity to image a certain number of distinguishable brightness levels of a random object set by the signal to noise ratio. The signal to noise ratio is derived from the ratio of the spectral power of the object over the noise power or Wiener spectrum of the granularity (Linfoot, 1964). Information capacity of a photographic image is derived as a function of the signal to noise ratio as follows:-

Let the signal to noise ratio be defined by

$$S(u,v) = \frac{T^2(u,v) \;.\; p(u,v)}{n(u,v)} \tag{10}$$

where $T(u,v)$ is the photograph film MTF

$p(u,v)$ is the power spectrum of the object and is independent of the imaging system

$n(u,v)$ is the Wiener spectrum of the granularity.

Therefore information capacity

$$I = B \iint_{-\infty}^{\infty} \log_2(1 + S(u,v))du\,dv \text{ bits/image} \tag{11}$$

where B is the emulsion area.

Information Theory states that the information content of a received signal (in this case the image) is maximized when the transmitted signal (the object) is random; that is, at the time of receiving the signal there is no fore-knowledge of its content. This is equivalent to assuming that $p(u,v)$ is constant p_o, over all frequencies. Studies for specific object characteristics, perhaps those occurring in photogrammetry, may be made assuming appropriate values of $p(u,v)$ which are not constant. However, this aspect will not be considered in this paper as it limits the generality of the treatment. Again assuming isotropic properties of the emulsion, equation (11) may be rewritten as:-

$$I = 2\pi \int_o^H \log_2(1 + \frac{T^2(w)p_o}{n(w)})w\,dw \text{ bits/unit area.} \tag{12}$$

where p_o is the constant object spectral power

H is the spatial frequency band width associated with the computation.

This formula does not measure the similarity between object and image as a computation involving the spread function or resolution may do. What it does do however, is derive a measure of performance of a photographic system as a communications system incorporating the effects of granularity. The similarities between this measure and visual performance must be considered separately.

Shaw (1962) rewrote equation (12) in terms of measured densities as follows, assuming a low contrast object set and film gamma γ:-

$$I = 2\pi \int_o^H \log_2(1 + \frac{0.189\gamma^2 T^2(w)p_o}{n(w)})w\,dw \text{ bits/unit area.} \tag{13}$$

If it may be assumed that $n(w)$ is constant over a range of frequencies, for example, $H<100$ lines/mm, which is indeed the case for aerial films, equation (13) may be reduced to:-

$$I = 2\pi \int_0^H \log_2 (1+ 0.189\gamma^2 T^2(w)R)wdw \text{ bits/unit area.} \qquad (14)$$

Information capacity is therefore maximized by improving the MTF T(w) or increasing R and perhaps film gamma provided it does not also magnify the noise component of R. As T(w) may also include other components of the imaging system (Linfoot 1964) as was done in Trinder (1978), a measure of the image quality of a complete photogrammetric system is obtainable using this method which will consider the effects of photographic development and granularity, as well as all components involved in the image formation. Computations of equation 14 for 3 film samples will be given in Section 6.

4. Measurement of Granularity

4.1 Equipment

A Joyce Lobel Double Beam Scanning microdensitometer was available for the measurement of granularity. Rectangular apertures were available in the instrument, while circular apertures were obtained by placing artificially produced circular openings in the measuring beam. Following the recommendations of Zweig (1956b) approximately 3,000 samples were derived on each trace at intervals of 1μm for Test 1 and one-fifth of the aperture size for Tests 2 to 5. As a ratio arm of 1:1,000 was used for Tests 2 to 4, and 1:500 for Test 5, these intervals were equivalent to 2.5mm to 5mm on the graphical trace. The recording table length is approximately 25cm, and hence a large number of successive traces were required in order to derive the requisite number of sample points. To achieve this, at the end of each scan line the recording table was unclamped and reversed while the sample table remained fixed. Before scanning a new line the recording pen was displaced to avoid an overlap of the traces; between 5 and 10 traces were normally recorded on each page. The total length of scan on the film sample varied from 7.5 to 30mm. Correction of systematic drift in the graphical trace relative to a fixed linear coordinate system was achieved by the computation of a high order polynomial (normally 8th order) to represent the mean density level $\overline{D}$, equation (1). Random variations in density above and below this datum were the components $(D_i - \overline{D})$ required for the computation of the RMS value and the correlogram. Residual systematic errors, small non-zero values, were revealed in the "tail" of the correlogram; the method of treatment of this tail is discussed in Section 5.

Special attention had to be paid to the focussing of the optics of the microdensitometer. It was found necessary to adjust the optics after each individual trace because of the limited field of view of the optics, and the slight non-parallelism of the focal plane of the optics and the plane in which the film sample traversed. The slightly less reliable results of Test 4 may be partly attributed to insufficient attention being paid to this factor.

4.2 Film Samples

The black and white aerial films tested were Kodak 2405 (Double-X) and Agfa-Gevaert Pan 33. Exposures through neutral density filters resulted in densities ranging from approximately 0.5 to 1.6. Development conditions were varied such that gamma values ranged from approximately 0.6 to 1.2. A summary of the films and conditions of development is given in Table 2. In addition, a number of samples prepared by the Central Mapping Authority of N.S.W. under operational conditions on an OPECO continuous processor were obtained. The range of densities of these samples was not as complete as in the other tests, but the information has been included for comparison.

TABLE 2

Test	Film	Gamma	Chemistry	Approx. Developing Time
		Prepared in U.N.S.W. Laboratories		
C	Kodak 2405	0.65	Kodak D76	8 min.
D	"	0.63	" "	6 min.
E	"	1.10	Kodak DK50	8 min.
F	"	0.95	" "	6 min.
G	"	1.24	" "	10 min.
H	Pan 33	0.97	Gevaert G5	4 min.
I	"	1.20	" "	6 min.
		Prepared in Photographic Laboratories of C.M.A.		
J	Kodak 2405	0.86	Gebaert G2 Developer	Speed 22IPM
K	"	1.08	"	" "
L	"	1.19	"	" "
M	"	≈ 1.3	Gevaert G2/G5 Mix.	" "

5. Computations

The RMS values and correlograms were computed from equations (1) and (3) from the reduced density readings following the polynomial correction for systematic drift, while the two-dimensional Fourier transform of the correlogram was computed for the determination of the Wiener spectrum. The maximum value of the Wiener spectrum was derived by computation of the definite integral of the correlogram, which is effectively the volume under the correlogram curve. Total noise power which is the volume under the Wiener spectrum curve is obtained from the maximum value of the correlogram, since the correlogram and Wiener spectrum are transform pairs.

The "tail" of the correlogram was not included in the volume computation which was limited to values above zero if the tail was in the negative range, or the point where the curve reached a constant positive value (Zweig, 1956b). A progressive accumulation of volume was printed so that the cut-off point on the curve could be varied if the correlogram revealed irregular characteristics. The abscissa coordinate together with the maximum spatial frequencies of the Wiener spectrum are dependent on the aperture diameter. Typical curves of both the correlogram and the Wiener spectrum are shown in Figure 1.

6. Presentation of Results

This project was aimed at investigating properties of granularity

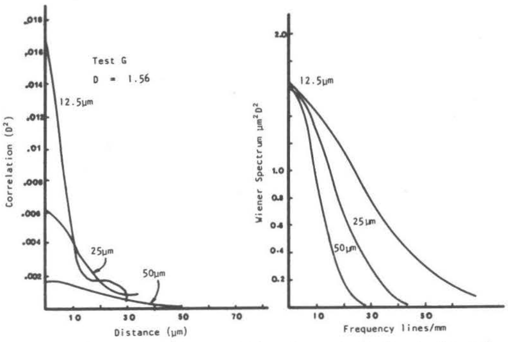

FIGURE 1: Typical Correlograms and Wiener Spectra for Chemistry G, D = 1.56

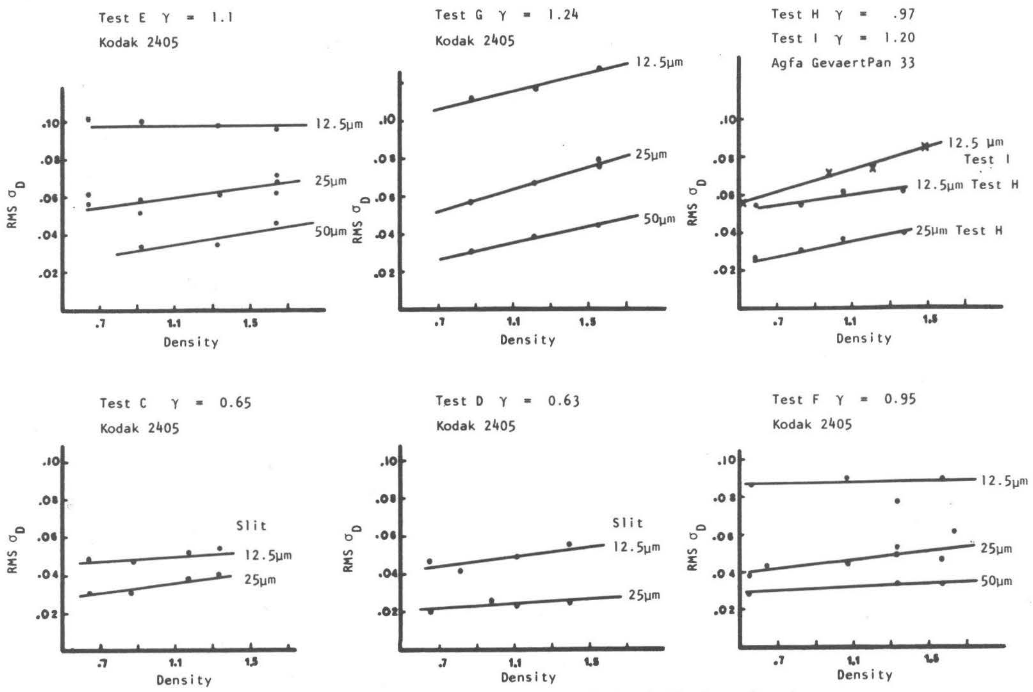

FIGURE 2: Relationship between RMS granularity and Density of Film Sample Chemistry C to I.

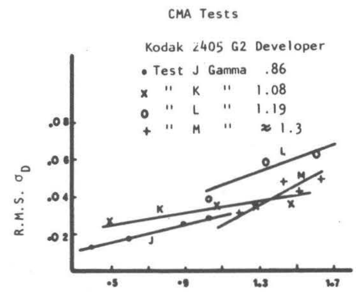

FIGURE 3: Relationship between RMS granularity and Density of Chemistry K to M.

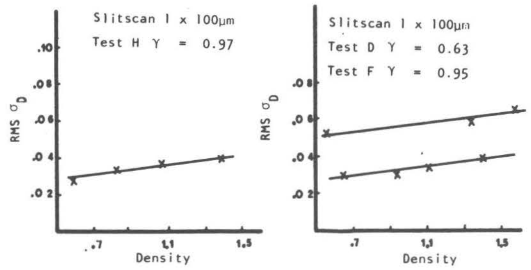

FIGURE 4

under the following general headings,

(i) Conformity of measured granularity to known properties of photographic emulsions
(ii) Wiener spectrum properties and image assessment involving Information Theory.

6.1 Properties of Emulsions

The RMS values have been plotted against density in Figures 2 to 4. Straight line relationships between RMS and density have been interpolated from these data. Accuracies of determination of RMS values are a function of the quality of the film samples, but more importantly on the mechanical operation of the microdensitometer, the major sources of inaccuracy being in the incorrect focussing of the optics, and the mechanical inaccuracies of the chart recorder. Attempts were made to overcome these effects, but residual errors may remain in the data. It is estimated that the standard deviation of RMS values is 0.002 to 0.003 D. Descrepancies between results of Tests 3 and 4 revealed in the plots for 25μm aperture are mainly caused by incorrect focussing of the optics during early stages of Test 4. A strong linear relationship between RMS values and density of sample is well established in these figures. This is consistent with characteristics of normal black and while emulsions.

In Figure 5 circular aperture diameter is plotted against RMS values at density 1.0 as interpolated from Figure 2. Lines parallel to the broken line satisfy the Selwyn relationship. There is strong agreement with this relationship for all lines plotted in Figure 5.

RMS values derived from Figures 2 to 4 at density 1.0 have been plotted in Figure 6 against film gamma for the aperture diameters applicable. These results reveal a substantial increase in the RMS values with increasing gamma and are consistent with Zwick (1965). This relationship appears to be independent of the developer used. Differences in behaviour of Kodak 2405 and Agfa-Gevaert Pan33 films are also revealed in Figure 6.

Relationships between RMS and respectively density and film gamma hold also for granularity measurements derived using rectangular apertures. Indeed, the slopes of corresponding relationships in all cases are substantially equal, the difference being only in the absolute magnitude of the RMS values.

6.2 Wiener Spectrum

Conceptually, the use of the Wiener spectrum as a measure of image quality is similar to using MTF's. Both are expressed in the frequency domain and the effects of different components of an imaging system, e.g. lens, film etc., can be combined relatively simply. The use of the Wiener spectrum however, has the advantage that effects of granularity may be combined with MTF's in a similar manner to that shown in equation (9), where M may be considered as the MTF of one or more components of a photographic system. In addition the propagation of the effects of granularity through the printing stages can be derived (Doerner, 1962). The phase component of an optical transfer function is lost when the Wiener spectrum is used, but it is generally considered that its effect is small.

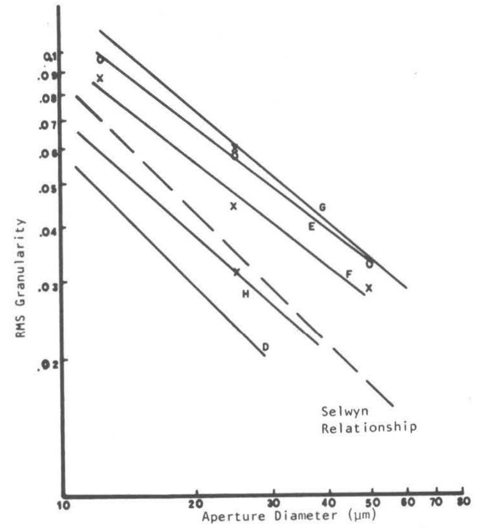

FIGURE 5: Relationship between aperture diameter and RMS granularity for film samples D to H. Lines parallel to the broken line demonstrate Selwyn's relationship for granularity.

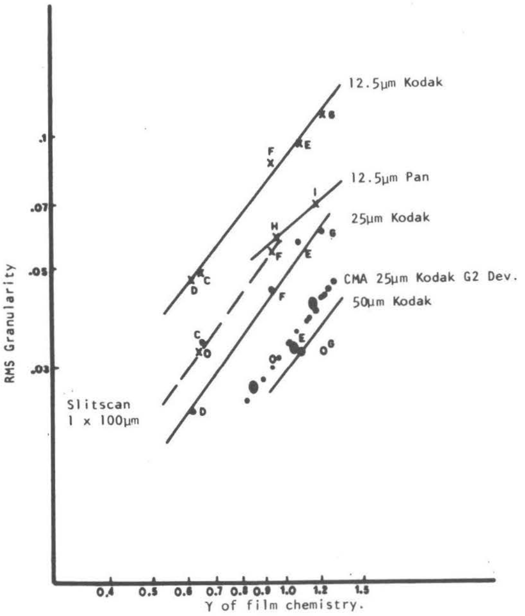

FIGURE 6: Relationship between RMS granularity at Density 1.0 and film γ for the 3 apertures of 12.5, 25 and 50μm (full lines) aperture of 1 x 100 m (broken line) and CMA film samples (dotted line).

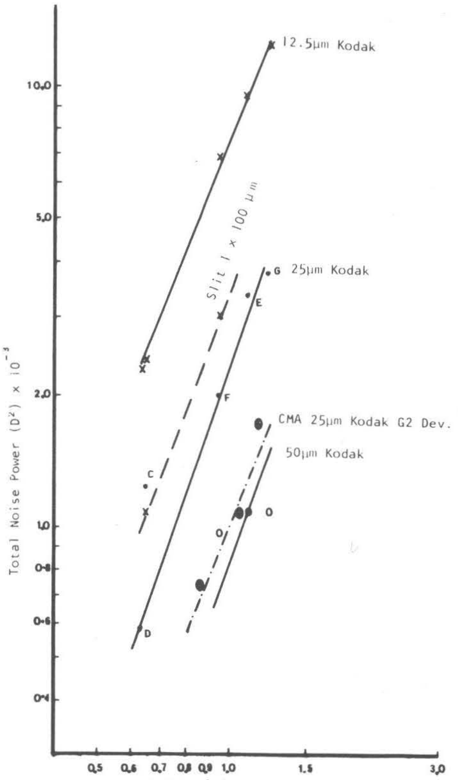

FIGURE 7: Total Noise Power (RMS²) v. Film Gamma

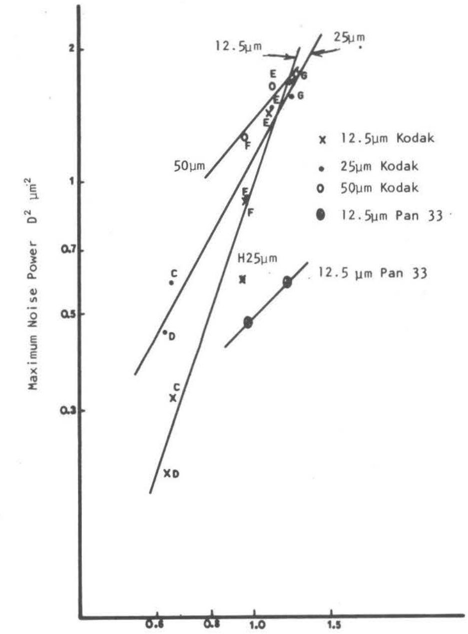

FIGURE 8: Film Gamma v. Maximum Noise Power.

A major difficulty associated with the use of MTF's or the Wiener spectrum is that of relating them to an observer's ability to perceive or measure to details. This requires an appreciation of graininess as well as an understanding of the importance of various spatial frequencies on visual performance. These aspects will be studied in phase two of this project.

Parameters of the Wiener spectrum of granularity which may prove to be suitable for assessing image quality are:-

(i) The cut-off frequency or frequency at which the power spectrum is zero. As shown in Figure 1 this point varies with aperture size but reveals nothing of the magnitude of granularity, and therefore is generally unsuitable. The effective cut-off frequencies of each aperture, which are significantly constant for all samples, are however important for future understanding of the effects of granularity.

(ii) Total Noise Power - definite integral of the Wiener Spectra and equal to $(\mathrm{RMS})^2$. This value is a summation of contributions at all spatial frequencies but no information is available on its actual shape. Variations of total noise power with gamma at 1.0 D are shown in Figure 7.

(iii) Maximum value at zero frequency - equal to the definite integral of the correlogram - is somewhat unreliable in its determination due to the existence of the "tail" in the correlogram. For circular apertures the maximum values are almost equal, as revealed in Figure 8, and therefore such values on their own are unsuitable parameters.

For rectangular slits a clear relationship exists between density and maximum noise power, as revealed in Figure 9. Noise power derived from rectangular slits is significantly constant for spatial frequencies less than 100 lines/mm.

These three parameters describe characteristics of the Wiener spectrum for both rectangular and circular apertures. However, while they describe individual aspects of the spectrum consideration of the signal to noise ratio involves all frequencies as shown in Equation 10. The significance of different circular apertures is especially revealed in the differences in the cut-off frequency, there being no effect on the signal to noise ratio for frequencies higher than the cut-off.

The characteristics of the granularity are more completely described by the power spectra derived using the rectangular aperture, where the effects of the slit have been substantially eliminated for frequencies less than 100 lines/mm. This data is therefore used for the computation of image assessment based on Information Theory.

7. Information Content of Films and Photographic Systems

Results of computations of information capacity (Equation 14) are presented in Figures 11 and 12. The signal to noise ratio $S(u,v)$, Figure 11, is constant for spatial frequencies less than 100 lines/mm; signal power p_0 is by definition constant and noise power is constant within experimental error. The total information capacity as defined by Equation (14) is plotted in Figure 12 for each of the 3 Tests D, F and H. In this case the dependence of information capacity on signal power is demonstrated (interpolated relationships are approximate only). Information capacity

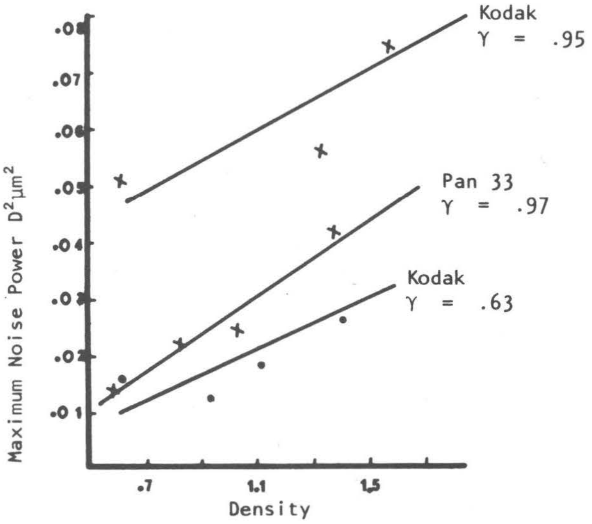

FIGURE 9: Relation between density and Maximum Noise Power Scan by rectangular slit 1 x 100μm.

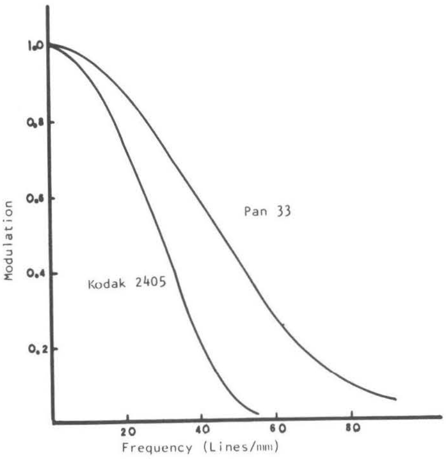

FIGURE 10: Measured MTF's of Kodak 2405 and Gevaert Pan 33 films.

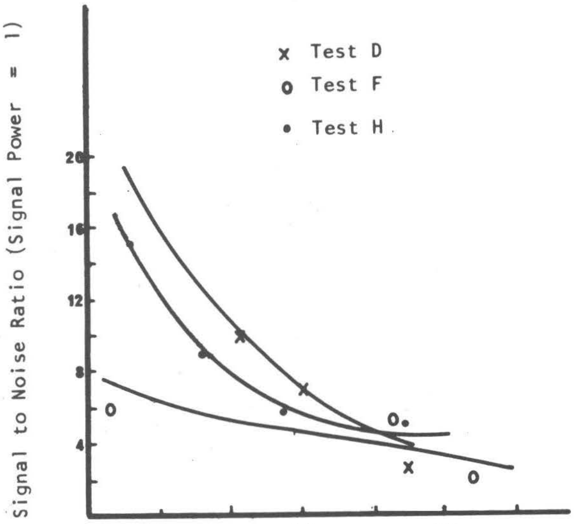

FIGURE 11

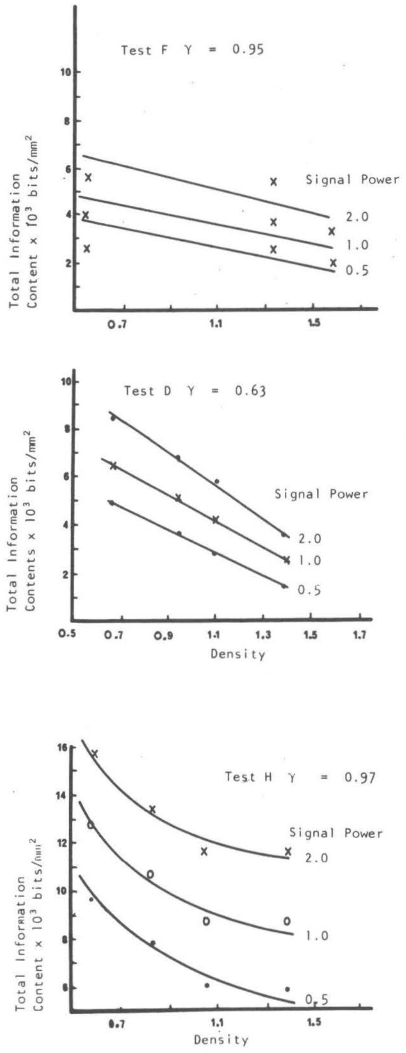

FIGURE 12

clearly decreases as noise power increases with increasing density, the decrease being also reflected in the signal to noise ratio. A comparison of films indicates that the better MTF of the PAN33 film (Test H), as shown in Figure 10, results in a higher information capacity than for the Kodak 2405 film. It is clear that MTF's have a major effect on information capacity, as indeed they also influence an observer's perception of fine details.

The application of data presented in Sections 5 and 6 to image assessment and observer performance will be the subject of investigation at a later stage in this research project. Information capacity and image quality are clearly correlated, but the impact of this method on assessing interpretability and measurability is yet to be determined, especially when different viewing magnifications are used.

Conclusions

1. The behaviour of the aerial film emulsions examined in this study in terms of aperture size, gamma and density is consistent with that reported for other black and white emulsions.

2. A number of parameters of the Wiener spectrum have been investigated for both circular and rectangular apertures. Those obtained for circular apertures are more applicable to problems of investigating graininess, while those obtained with rectangular apertures describe the 2-dimensional Wiener spectrum of the emulsion.

3. Studies reveal the importance of considering the signal to noise ratio in attempting to relate visual performance to image quality parameters.

Acknowledgements

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