



## **An Integrated Multi-Objective Mathematical Model to Select Suppliers in Green Supply Chains**

P. Mohammad<sup>1</sup>, H. Kazemipoor<sup>2\*</sup>

<sup>1</sup> Department of Industrial Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran.


<sup>2</sup> Faculty of Engineering, Parand Branch, Islamic Azad University, Tehran, Iran.

(\*Corresponding Author's Email Address: hkazemipoor@yahoo.com)

### **ABSTRACT**

Compatibility with the environment is one of the important factors in designing a supply chain system, which is also called the “green supply chain”. Similar to the supply chain, green suppliers are very important players in the green supply chain. This paper studies both selection of suppliers and optimal order allocation to them. Despite previous studies, we consider both strategic and operational decisions into the problem. Firstly, we investigate the relevant criteria in selecting suppliers, and assign appropriate weights to suppliers. Then, we apply the fuzzy TOPSIS technique to assess and rank the suppliers. Finally, we investigate optimal allocation of order to the suppliers. For this reason, a two-objective mathematical model is developed. To solve the model, “weighting” and “ $\epsilon$ -constraint” methods will be investigated, followed by a sensibility analysis to study the changes in the problem’s parameters. The proposed approach is important because it models the strategic and operational decision simultaneously.

**Keywords:** Green supply chain management, Green supplier selection, Optimal order allocation, Multi-objective supplier selection.

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## **1. Introduction and Problem Statement**

A supply chain includes steps, i.e. parts of the chain, which play a role in customer's satisfaction, either directly or indirectly. In a typical supply system, suppliers send the raw materials to factories. Then, the factories send their products to intermediate stores and/or distributor sites, and from there, the products head to retailers, and finally to customers or consumers. It has been more than two decades that the green supply chain management has been studied. Many companies and industries are keen to initiate a partnership with the suppliers, so they would be able to enhance their competitiveness performance globally. Continuity and stability of the

relationship between suppliers, companies and industries may lead to the supply chain of the company to be a serious bottleneck to the competitors [1]. Moreover, a long-term and stable relationship with the supplier may lead to reduction in the costs of the supplier, which in turn may result in reducing the costs of the organization, i.e. mutual benefits. Also, a stable relationship leads to the situation, in which suppliers follow the policies, standards and terms and conditions of the employer, and will use the available facilities of suppliers.

Meeting customers' demands must be considered by the producers and suppliers, as well as those provide raw materials, manufacturing, assembly, packaging, distribution, and finally marketing. In this regard, we are facing with many significant decisions, which have a high level of complications.

Dickson suggested 23 different criteria to evaluate the performance of suppliers and providers [2]. The quality, on-time delivery and the history and background have been recognized as three important criteria. Topcu studies a multi- criteria decision model to select construction suppliers in Turkey [3]. Bagheri and Tarokh proposed a multi-objective model, including hierarchical planning and Fuzzy TOPSIS to select suppliers [4]. Toloo and Nalchigar presented a new model of integrated data development analysis, which is able to identify the most effective provider by using the qualitative and quantitative data [5].

Another important matter in the supply chain includes integrating decision making about selecting the suppliers with the relationship among them, and the level of optimal order. In this regard Lin in 2009 proposed two-step approach to select the suppliers [6]. The approach combines multi-criteria and multi-objective decisions. Later, Lin in 2012 developed an integrated Fuzzy Analytic Network Process (FANP) with a multi-objective linear program [7]. A multilevel model of price and product has been suggested to supplier selection and their optimal allocation order [8]. The author, in contrary to the model of Lin [6], investigated discount on the purchase of goods and multi production ability. The author presented criteria such as minimization of total cost, minimization of the purchase of defective products, and minimization of product delivery time, and used a fuzzy approach to solve the proposed goal programming model. In addition, to analyze the sensitivity of suggested model, the study applied five scenarios based on the purpose functions. The results of the scenarios show the validity of the models. Kannan et al. presented a combined approach based on FANP, fuzzy TOPSIS and multi-objective linear programming in order to select suppliers in a green supply chain [9]. Shaw et al., presented an integrated approach for selecting the appropriate supplier in the supply chain, addressing the carbon emission issue, using fuzzy-AHP and fuzzy multi-objective linear programming; see also, [11]. Arabzad [12], also combined multi-criteria and multi-objective decision making in order to select the suppliers, as well as determining the optimal orders. Ware et al., presented a mixed integer nonlinear model for supplier selection in a dynamic environment [13]. Likewise, the study by Cui looked into supplier selection by considering production planning [14]. Razmi et al. suggested a goal programming approach to solve multi-objective programming problems in similar environments [15]; for other recent works see [16-20].

The present research aims to overcome the current shortcomings, and contributes to the research on the green supply chain selections by proposing a two-step approach to assess and select the suppliers, considering traditional and green criteria at the same time. The first step, which is a strategic step, assesses the eligibility of suppliers by using a set of critical criteria for organizations. The eligible suppliers enter to the second step, which is an operational step, i.e. decisions such as working or not working with the suppliers, optimal supply routes, etc. are made at this step. For the first step, we proposed a multi-criteria decision-making model, and for the second step we proposed a multi-objective mathematical program. We also discuss solving the models.

The remaining of the paper is organized as follows. In Section 2, we discuss the two-step approach. Section 3 details the mathematical model for selecting suppliers and allocating orders. In Section 4, the implementation of the suggested approach will be explained, followed by an example and discussions in Section 5. Finally, conclusions are discussed in Section 6.

## **2. The Proposed Approach**

We developed several criteria in order to evaluate and rank suppliers. We also consulted with the previous studies in order to further complete the set of criteria. Each criterion of the set is further narrowed down into a set of sub-criteria. Then, experts are asked to assign weights to the sub-criteria. Because we may encounter ambiguity in experts' opinions, therefore, we used the fuzzy TOPSIS technique in order to overcome this ambiguity, and to weight (rank) the suppliers. Given the weights of suppliers, optimal allocation of orders to the suppliers can be obtained. We chose a TOPSIS -based model because it is a well-established technique, and the method selects an alternative, which has the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution.

In many organizations evaluating the performance of suppliers, and selecting them and order allocation take place in two steps. First, the pre-eligibility of suppliers is assessed against the minimum requirements. Those suppliers that pass the first step enter into the second step. The second step takes into account important factors such as delivery capacity, delivery time, quality, cost, etc., and makes operational decisions, for example, order quantity of each supplier, routing and delivery planning, and pricing, among others.

### **2.1. Two-Step Approach**

This study also proposes a two-step approach: Step 1 evaluates suppliers (strategic decision) by using a fuzzy TOPSIS -based method, and Step 2 develops a two-objective mathematical model in order to obtain optimal allocation of orders to suppliers (operational decision).

**Step 1. selecting and evaluating suppliers**

In Step 1, we extract criteria and sub-criteria involved in selecting suppliers, assign weights to the sub-criteria, and evaluate the performance of each supplier per each sub-criteria.

**Assigning weights to the sub-criteria**

Firstly, weights are assigned to the sub-criteria, and then weights are assigned to each metric. We ask a group of experts to assign weights to sub-criteria. Given a group of  $k$  experts  $(D_1, D_2, \dots, D_k)$ , and  $m$  evaluation metrics, which will be compared against each other by using  $n$  criteria  $(C_1, C_2, \dots, C_n)$ , the experts are asked to use Table 1 for evaluating the relative importance of sub-criteria .and assign weights to the criteria.

**Table 1.** The linguistic expressions and their triangular fuzzy numbers.

Linguistic expressions	Triangular fuzzy numbers
Very low	(0, 0.2, 0.4)
Low	(0.2, 0.4, 0.5)
Medium	(0.4, 0.6, 0.8)
High	(0.6, 0.8, 1.0)
Very high	(0.8, 0.9, 1.0)

Then, we de-Fuzzy the triangle Fuzzy numbers by using Equation (1).

$$\frac{a + 4b + c}{6} \tag{1}$$

where,  $a$ ,  $b$ , and  $c$  are three parameters (low, middle and top) of the triangle Fuzzy number. This results in  $W'_{jt}$ , the weight of sub-criterion  $j$  by expert  $t$ . Equation (2) calculates the weight of each sub-criterion.

$$W_j = \frac{\sum_{t=1}^k W'_{jt}}{k} \tag{2}$$

Secondly, experts are asked to use Table 2 and assign weights to each metric and per sub-criterion. Then, we apply the Fuzzy TOPSIS, and assign weights to every sub-criterion.

**Table 2.** Linguistic variables and fuzzy rating indicators.

Linguistic expressions	Triangular fuzzy numbers
Very weak	(0, 0, 1)
Weak	(0, 1, 3)
Medium-weak	(1, 3, 5)
Medium	(3, 5, 7)
Medium-high	(5, 7, 9)
High	(7, 9, 10)
Very high	(9, 10, 10)

**Evaluating the performance of suppliers**

This step ranks the suppliers by using questionnaire. In fact, experts are asked to apply Table 2 and score the factors per each supplier. Triangular Fuzzy numbers associated with these linguistic terms are shown in Table 3.

*Table 3.* Linguistic terms used in evaluating supplier’s performance.

Favorable linguistic terms	Unfavorable linguistic terms	Equivalent triangle fuzzy numbers	The mean of triangle fuzzy numbers
Very weak	Very good	(0,0,0)	0
Weak	Good	(0,0.167,0.333)	0.167
Medium to weak	Medium to good	(0.167,0.333,0.5)	0.333
Medium	Medium	(0.333,0.5,0.667)	0.5
Medium to good	Medium to weak	(0.5,0.667,0.833)	0.667
Good	Weak	(0.667,0.833,1)	0.833
Very good	Very weak	(1,1,1)	1

Then, the average of experts' opinion will be calculated for each sub-criterion and every supplier. To determine the final score of each supplier, total product of criteria weights by their values will be calculated (we calculate the total product of weight factors and their numerical values). These scores will be utilized as the objective function coefficients (of one of the objectives) in the mathematical model of Section 4.

**Step 2. Allocating orders**

The suppliers with the highest score enter into Step 2, in which decisions on the amount of products to be received from each supplier are made. We model this decision making problem as a two-objective mixed integer non-linear program. This model is based on the following assumptions:

- The supply chain only includes supply and sale levels. Moreover, the supply chain is multi-period.
- Geographic location of suppliers and sale rooms are given (they are not determined by the model).
- The number of vehicles, which transport items from suppliers to sale locations, and their capacity are known.
- Travel times of vehicle are known, and are deterministic. Also, we assume that vehicles are not permitted to break.
- Items may be stored at sale locations. Also, items may be transferred into future periods.

**2.2. Mathematical Notations**

We have five indices in the proposed model. These indices represent items( $i = 1, \dots, N$ ), suppliers ( $s = 1, \dots, S$ ), customers ( $c = 1, \dots, C$ ), vehicles ( $v = 1, \dots, V$ ), and time periods ( $t = 1, \dots, T$ ). Table 4 shows the parameters of the model.

The model decides upon the followings:

- Suppliers selection
- Using (or not using) a vehicle
- Amount of items transported by a vehicle
- Routing of the vehicles (sequence of picking up items)
- Amount of items to be stored

**Table 4.** The parameters of the model.

Parameter	Explanation
$dem_{tct}$	Demand of customer $c$ for item $i$ at time $t$
$cap_{ist}^{sup} \left  \sum_{i,t} cap_{ilt}^{sup} = 0 \right.$	Capacity of supplier $s$ in supplying item $i$ at time $t$
$cap_v^{veh}$	Capacity of vehicle $v$
$cap_c^{inv}$	Storage capacity of customer $c$
$Vol_i$	Amount of item $i$
$w_i$	Weight of item $i$ , $1 \leq w_i \leq 1$ .
$d_{cs}^{stc}$	Distance of supplier $s$ from customer $c$
$d_{s\hat{s}} \left  \sum_s d_{s1} = \sum_s d_{1s} = 0 \right.$	Distance of supplier $s$ from suppliers $\hat{s}$
$tm_{cs}^{st}$	Distance of supplier $s$ from customer $c$
$tm_{s\hat{s}} \left  \sum_s tm_{s1} = \sum_s tm_{1s} = 0 \right.$	Travelling time from supplier $s$ to suppliers $\hat{s}$
$c_v^{veh}$	Cost of acquiring vehicle $v$
$c_{ist}^{buy}$	Cost of buying a unit of item $i$ from supplier's $s$ at time $t$
$c_{ic}^{inv}$	Storage cost of a unit of item $i$ at the storage of customer $c$
$\eta_v$	Fuel consumption per unit of distance for vehicle $v$
$\hat{\eta}_v$	Fuel consumption per unit of distance for vehicle $v$ for an additional unit of item
$c^{feul}$	Fuel cost per unit of fuel
$\lambda_s$	Score of supplier $s$ , which is obtained in Step 1, $0 \leq \lambda_s \leq 1$
$M$	A very large positive constant

### 2.3. Decision Variables

The decision variables include

$z_{vt} \in \{0,1\}$ , which takes 1 if vehicle  $v$  is selected at time  $t$ , and 0 otherwise.

$\beta_{cvt} \in \{0,1\}$ , which takes 1 if vehicle  $v$  is allocated to customer  $c$  at time  $t$ , and 0 otherwise.

$x_{\hat{s}svt} \in \{0,1\}$ , which takes 1 if vehicle  $v$  leaves supplier  $\hat{s}$  to reach supplier  $s$  at time  $t$ , and 0 otherwise.

$y_{isvt} \in \mathbb{Z}^+$ , which is the number of received items  $i$  from supplier  $s$  at time  $t$  by vehicle  $v$ .

$inv_{ict} \in \mathbb{Z}^+$ , which is the number of item  $i$  stored at the location of customer  $c$  at time  $t$ .

$\alpha_{ict} \geq 0$ , which is the net amount of unspoiled items  $i$  shipped to customer  $c$  at time  $t$ .

$mas_{svt} \geq 0$ , which is the weight of items in vehicle  $v$  at the time of leaving supplier  $s$  at time  $t$ .

$at_{svt} \geq 0$ , which is the arrival time of vehicle  $v$  at the location of supplier  $s$  at time  $t$ .

$pc_t \geq 0$ , which is the price if purchasing items at time  $t$ .

$tc_t \geq 0$ , which is the price of transporting items at time  $t$ .

$hc_t \geq 0$ , which is the storage cost of items at time  $t$ .

### 2.4. Constraints and Objective Functions

The major constraints of the model include

- Capability of a supplier in supplying items
- Vehicle capacity
- Time windows for delivery (vehicles service times)
- Vehicle routing constraint
- Demand
- Storage capacity

The objective function includes two parts: 1) minimizing the costs associated with the supply chain, including costs of acquiring vehicles, material, transportation, and of storage, and 2) maximizing the suppliers score.

### 3. A Two Objective Mathematical Model

As discussed earlier, Step 2 can be modeled as a two objective mixed integer non-linear mathematical model, and may be presented as Model I.

#### Model I

$$\begin{aligned}
 \text{Min } obj_1 = & \sum_{i,c,t} C_{ic}^{inv} \times inv_{ict} + \sum_{i,s,v,t} C_{ist}^{buy} \times y_{isvt} + \sum_{v,t} C_v^{pur} \times z_{vt} + \\
 & \sum_{s,\hat{s},v,t} c^{feul} \times (\eta_v + \hat{\eta}_v \times mas_{svt}) \times d_{\hat{s}s} \times x_{\hat{s}svt} + \sum_{c,s,v,t} c^{feul} \times ((\eta_v + \hat{\eta}_v \times mas_{svt}) \\
 & \times d_{cs}^{stc} \times \eta_v + \times d_{cs}^{stc} \times x_{1svt} \times \beta_{cvt}
 \end{aligned} \tag{3}$$

$$\text{Max } obj_2 = \sum_{i,s,v,t} y_{isvt} \lambda_s \tag{4}$$

Subject to

$$\sum_c \beta_{cvt} \leq Z_{vt}^{veh} \quad \forall v, t \quad (5)$$

$$\sum_i y_{isvt} \leq M \sum_{\hat{s}} x_{\hat{s}svt} \quad \forall s, v, t \quad (6)$$

$$\sum_{\hat{s}, v} x_{\hat{s}svt} \leq 1 \quad \forall s, t \quad (7)$$

$$\sum_{\hat{s}} x_{\hat{s}svt} = \sum_{\hat{s}} x_{s\hat{s}vt} \quad \forall s, v, t \quad (8)$$

$$at_{svt} \geq \sum_{\hat{s}} (at_{\hat{s}vt} + tm_{\hat{s}s}) \times x_{\hat{s}svt} \quad \forall v, t, s > 1 \quad (9)$$

$$at_{svt} \geq \sum_c tm_{cs}^{stc} \times x_{1svt} \times \beta_{cvt} \quad \forall v, t, s \quad (10)$$

$$mas_{svt} \geq \sum_{\hat{s} > 1} mas_{\hat{s}vt} \times x_{\hat{s}svt} + \sum_i y_{isvt} \times w_i \quad \forall v, t, s > 1 \quad (11)$$

$$\sum_v y_{isvt} \leq cap_{ist}^{sup} \quad i, s, t \quad (12)$$

$$\sum_{i,s} y_{isvt} \times vol_i \leq cap_v^{veh} \times Z_{vt} \quad \forall v, t \quad (13)$$

$$\sum_{i,s} inv_{ict} \times vol_i \leq cap^{inv} \quad \forall c, t \quad (14)$$

$$\alpha_{ict} \leq \sum_{s,v} y_{isvt} \times \beta_{cvt} \quad \forall i, c, t \quad (15)$$

$$\alpha_{ict} + inv_{ic(t-1)} \leq \sum_{\hat{t}=t}^{t+\lambda_i} dem_{ic\hat{t}} \quad \forall i, c, t > 1 \quad (16)$$

$$\alpha_{ict} \leq \sum_{\hat{t}=1}^{1+\lambda_i} dem_{ic\hat{t}} \quad \forall i, c \quad (17)$$

$$\alpha_{ict} + inv_{ic(t-1)} - dem_{ic\hat{t}} = inv_{ic\hat{t}} \quad \forall i, c, t > 1 \quad (18)$$

$$\alpha_{ic1} - dem_{ic1} = inv_{ic1} \quad \forall i, c \quad (19)$$

$$Z_{vt} \in \{0,1\} \quad (20)$$

$$\beta_{cvt} \in \{0,1\} \quad (21)$$

$$x_{\hat{s}svt} \in \{0,1\} \quad (22)$$

$$y_{isvt} \in \mathbb{Z}^+ \quad (23)$$

$$inv_{ict} \in \mathbb{Z}^+ \quad (24)$$

$$\alpha_{ict} \geq 0 \quad (25)$$

$$mas_{svt} \geq 0 \quad (26)$$

$$at_{svt} \geq 0 \quad (27)$$

$$pc_t \geq 0 \quad (28)$$

$$hc_t \geq 0 \quad (29)$$

$$hc_t \geq 0. \quad (30)$$

The first objective function (Equation (3)) minimizes the total cost of supply chain, and the second objective function (Equation (4)) maximizes the suppliers' scores. Constraints (5) ensure a vehicle



is allocated to a customer. Constraints (6) state only those vehicles visiting customers can transport items. Constraints (7) state that in each time period every supplier can be visited by only one vehicle. Constraints (8) imply that if a vehicle arrives at a supplier’s location it must leave the location, where the arrival time is modeled via Constraints (9) and (10). According to Constraints (10), a vehicle visits a supplier only if it is available in the associated time period. Constraints (11) models the weight of transported items by vehicles when leaving the supplier’s location. The supplier, vehicle and storage capacities have been modeled by Constraints (12), (13) and (14). Constraints (15), (16) and (17) satisfy the condition on  $\alpha_{ict}$ , which is the amount of delivered items to customers (obviously, before the expiry date). Constraints (18) and (19) model inventory at the customer storage; we call these constraints the “inventory balance” constraints. Finally, Constraints (23) to (30) state the type of decision variables.

**4. Linearization**

Notice that in the proposed model (Mode I) Equations (1), (9), (10), (11), and (15) are non-linear. Following the computational difficulty in solving non-linear models, we aim to linearize those non-linear equations. For this reason, a set of auxiliary variables are needed. Table 5 shows those variables.

**Table 5.** Variables for linearization and their definition.

Auxiliary variables	Range	Auxiliary variables definition
$\beta y_{icsvt}$	Non-Positive	$\beta_{cvt} \times y_{isvt}$
$f_{svt}^1$	Non-Positive	$\sum_s (\eta_v + \hat{\eta}_v \times mas_{svt}) \times d_{\hat{s}s} \times \hat{s}_{svt}$
$f_{svt}^2$	Non-Positive	$\sum_c (\eta_v + \hat{\eta}_v \times mas_{svt}) \times d_{cs}^{stc} \times x_{s1vt} \times \beta_{svt}$
$f_{svt}^3$	Non-Positive	$\sum_c \eta_v \times d_{cs}^{stc} \times x_{s1vt} \times \beta_{svt}$

Let start by Constrains (9), which can be represented as linear Constraints (31):

$$at_{\hat{s}vt} + M \times (1 - x_{\hat{s}vt}) \geq at_{svt} + tm_{\hat{s}s} \quad \forall v, t, \hat{s}, s > 1 \tag{31}$$

Constraints (10) can be re-written as Constraints (32):

$$at_{svt} + M \times (2 - x_{1svt} - \beta_{cvt}) \geq tm_{cs}^{stc} \tag{32}$$

Constraints (11) can be replaced by linear Constraints (33):

$$mas_{svt} + M \times (1 - x_{\hat{s}svt}) \geq mas_{\hat{s}vt} + \sum_i y_{isvt} \times w_i \quad \forall v, t, \hat{s} > 1, s > 1 \quad (33)$$

We need Constraints (34), (35), and (36) in order to linearize Constraints (15):

$$\beta y_{icsvt} \leq y_{isvt} \quad \forall i, c, s, v, t \quad (34)$$

$$\sum_{i,s} \beta y_{icsvt} \leq M \times \beta_{cvt} \quad \forall c, v, t \quad (35)$$

$$\alpha_{ict} \leq \sum_{s,v} \beta y_{icsvt} \quad \forall i, c, t \quad (36)$$

Finally, Constraints (21) can be linearized by introducing Constraints (40) to (43).

$$tc_t = C^{feul} \times \sum_{s,v} (f_{svt}^1 + f_{svt}^2 + f_{svt}^3) \forall t \quad (37)$$

$$f_{svt}^1 + M \times (1 - x_{\hat{s}svt}) \geq (\eta_v + \hat{\eta}_v \times mas_{svt}) \times d_{\hat{s}s} \quad \forall s, v, t \quad (38)$$

$$f_{svt}^2 + M \times (2 - x_{s1vt} - \beta_{cvt}) \geq (\eta_v + \hat{\eta}_v \times mas_{svt}) \times d_{cs}^{stc} \quad \forall s, v, t \quad (39)$$

$$f_{svt}^3 + M \times (3 - x_{1svt} - \beta_{cvt}) \geq \eta_v \times d_{cs}^{stc} \quad \forall s, v, t \quad (40)$$

At the end of this process, we have a two-objective linear model, which we named it Model II.

## Model II

Objective functions (3) and (4)

Subject to

Constraints (5)-(8), (12)-(14), (16)-(21), and (31)-(40).

Section 5 discusses the methods to solve Model II through an illustrative example.

## 5. An illustrative Example

In this section we solve a set of eight randomly generated instances to further investigate our proposed model (Model II). All instances include six suppliers. We considered a set of 18 sub-criteria in order to rank suppliers.

### Step 1. Selecting and Evaluating Suppliers

As discussed earlier, Step 1 identifies and extracts criteria and sub-criteria for evaluating the suppliers, assigns weights to the sub-criteria, and evaluates the performance of suppliers. Table 6 shows those extracted criteria and sub-criteria. In total, we have three criteria of “Quality”, “On time delivery”, and “Green”, which are further narrowed down into 18 sub-criteria.

**Table 6.** The criteria and sub-criteria of evaluating the suppliers.

Notation	Criteria	Sub-criteria
S1	Quality	Applying quality control system
S2		The previous customers satisfaction
S3		The quality of after-sales service
N1	On time delivery	Project control system and efficient production
N2		Program to deal with delay
N3		Mechanisms to reduce process time
N4		Written materials for planning system
N5		Customer orders planning system
N6		Appropriate risk analysis system
C1	Green	Air pollution
C2		Compliance with environmental standards (such as ISO 14000)
C3		Buy ecosystem-friendly raw materials
C4		Eco-partners
C5		Green packing
C6		Recovery (waste reprocessing reusable materials)
C7		Access to clean technology for reverse logistics
C8		Sustainable Design
C9		Energy consumption

Table 7 shows weighs of every sub-criterion per supplier by using the Fuzzy TOPSIS method discussed earlier. The last row of Table 6 shows the final score of each supplier. The calculation for obtaining the final scores was explained in Section 3.3.

**Step 2. Allocating Orders**

In order to validate Model II, we generated a set of eight random instances with different sizes and parameters, and solved Model II. For these instances, the values of parameter  $\lambda_s$  are the final scores of Table 6. Model II was implemented in the modeling language GAMS version 24.1, and was solved by using the standard solver CPLEX. Recall that because Model II includes two objective functions, we utilize two approaches of “weighting” and “ $\epsilon$ -constraint” methods in order to derive a single objective optimization model. The procedure of building the eight randomly generated instances was implemented in the computational package MATLAB Version 2012.

The parameters of the generated instances have been illustrated in Table 8. The first column is the name of instances. The remaining columns show the number of items ( $i$ ), the number of suppliers ( $s$ ), the number of customers ( $C$ ), the number of vehicles ( $v$ ), and the number of time periods ( $T$ ).

**Table 7.** The final score of suppliers.

Sub-criterion	Supplier					
	1	2	3	4	5	6
S1	0.0115	0.0173	0.0326	0.0230	0.0268	0.0173
S2	0.0175	0.0222	0.0206	0.0206	0.0143	0.0175
S3	0.0181	0.0336	0.0259	0.0284	0.0336	0.0284
N1	0.0175	0.0154	0.0285	0.0307	0.0374	0.0154
N2	0.0073	0.0091	0.0237	0.0146	0.0219	0.0146
N3	0.0394	0.0344	0.0492	0.0492	0.0344	0.0295
N4	0.0208	0.0185	0.0300	0.0196	0.0208	0.0208
N5	0.0285	0.0338	0.0374	0.0321	0.0338	0.0321
N6	0.0356	0.0375	0.0356	0.0375	0.0431	0.0412
C1	0.0293	0.0204	0.0191	0.0153	0.0178	0.0306
C2	0.0218	0.0232	0.0245	0.0205	0.0245	0.0273
C3	0.0279	0.0262	0.0419	0.0315	0.0349	0.0419
C4	0.0282	0.0373	0.0321	0.0447	0.0495	0.0242
C5	0.0257	0.0265	0.0502	0.0330	0.0574	0.0502
C6	0.0191	0.0151	0.0295	0.0381	0.0193	0.0244
C7	0.0196	0.0316	0.0265	0.0304	0.0403	0.0345
C8	0.0340	0.0400	0.0237	0.0455	0.0347	0.0455
C9	0.0115	0.0173	0.0326	0.0230	0.0268	0.0173
Final score	0.4134	0.4593	0.5637	0.5376	0.5716	0.5126

**Table 8.** Size and parameters of the eight randomly generated instances.

Instance	Parameters				
	N	S	C	V	T
P1	2	3	3	5	3
P2	2	4	3	6	3
P3	3	4	3	6	3
P4	3	4	4	7	3
P5	3	5	4	7	4
P6	4	5	4	8	4
P7	4	6	5	9	4
P8	4	7	5	10	4

### 5.1. Solving model II by using the weighting method

In order to optimize and solve the two-objective linear model presented in Model II, we can use the common approach of weighting the objective function elements. Because the objective function of

Model II has two elements with different scales, we need to ensure they have a similar scale. For this reason, we apply the non-scaling method:

$$MinZ = w_1 \left( \frac{f_1 - f_1^+}{f_1^- - f_1^+} \right) + w_2 \left( \frac{f_1^+ - f_2}{f_2^+ - f_2^-} \right) \tag{41}$$

where,  $w_1$  and  $w_2$  are the weights of objective functions  $f_1$  and  $f_2$ . The weights can be obtained from experts and decision makers of the field. In Equation (44)  $f^-$  and  $f^+$  represents the worst and the best values for the objective function, and indices 1 and 2 refer to objective functions 1 and 2. To obtain  $f^-$  and  $f^+$  we solve Model II, however, by optimizing one of the objective functions at a time. The outcomes can be shown by a square  $2 \times 2$  matrix, which is shown in Equation (42) (generally, if  $n$  objective functions exist, then the outcomes are a  $n \times n$  matrix).

$$\begin{pmatrix} f_1^+ & f_2^- \\ a_{21} & f_2^+ \end{pmatrix} \tag{42}$$

Table 9 shows the outcomes of standard solver CPLEX. It can be seen that CPLEX is quite able to solve all eight instances, and that in a reasonable amount of time. Even for the most challenging instances, i.e. ‘‘P8’’, the solver CPLEX obtains the optimal solution within 32 minutes.

Figure 1 illustrates the performance of solver CPLEX for the first and the second objective functions, along with the lower and upper bounds. According to Figure 1, increasing the number of items, customers, and time periods leads to an increase in the cost (the value of the objective function). Because when more items should be delivered, more operations are involved, e.g. more trips by the vehicles. However, if we only increase the number of vehicles and suppliers, the total cost should not increase. Indeed, by a closer look into instances ‘‘P1’’ and ‘‘P2’’, and ‘‘P7’’ and ‘‘P8’’ one may conclude this.

**Table 9.** Computational results of solving the eight random instances by utilizing the weighting method, and solving by the standard solve CPLEX.

Instance	$f_1^-$	$f_1^*$	$f_1^+$	$f_2^+$	$f_2^*$	$f_2^-$	$z$	Time(s)
P1	522247	471264	431001	4052.25	3609.85	2933.34	0.418	18.21
P2	529787	427634	370069	4696.75	3717.16	2920.51	0.456	31.91
P3	606082	454147	402314	4746.89	3843.48	3347.11	0.450	44.15
P4	667989	609239	488957	5465.16	4094.06	3428.82	0.673	161.88
P5	795790	687404	611587	6759.12	5278.21	4515.24	0.536	419.06
P6	903234	751147	625952	6936.17	5836.59	5101.45	0.525	1119.99
P7	1080185	840759	702845	7802.92	6601.49	5430.65	0.436	1268.38
P8	979767	822598	699373	8494.81	6651.95	5853.61	0.569	1898.91
Min	522247	427634	370069	4052.25	3609.85	2920.51	0.418	18.211
Mean	760635	633024	541512	6119.26	4954.10	4191.34	0.508	620.31
Max	1080185	840759	702845	8494.81	6651.95	5853.61	0.673	1898.91

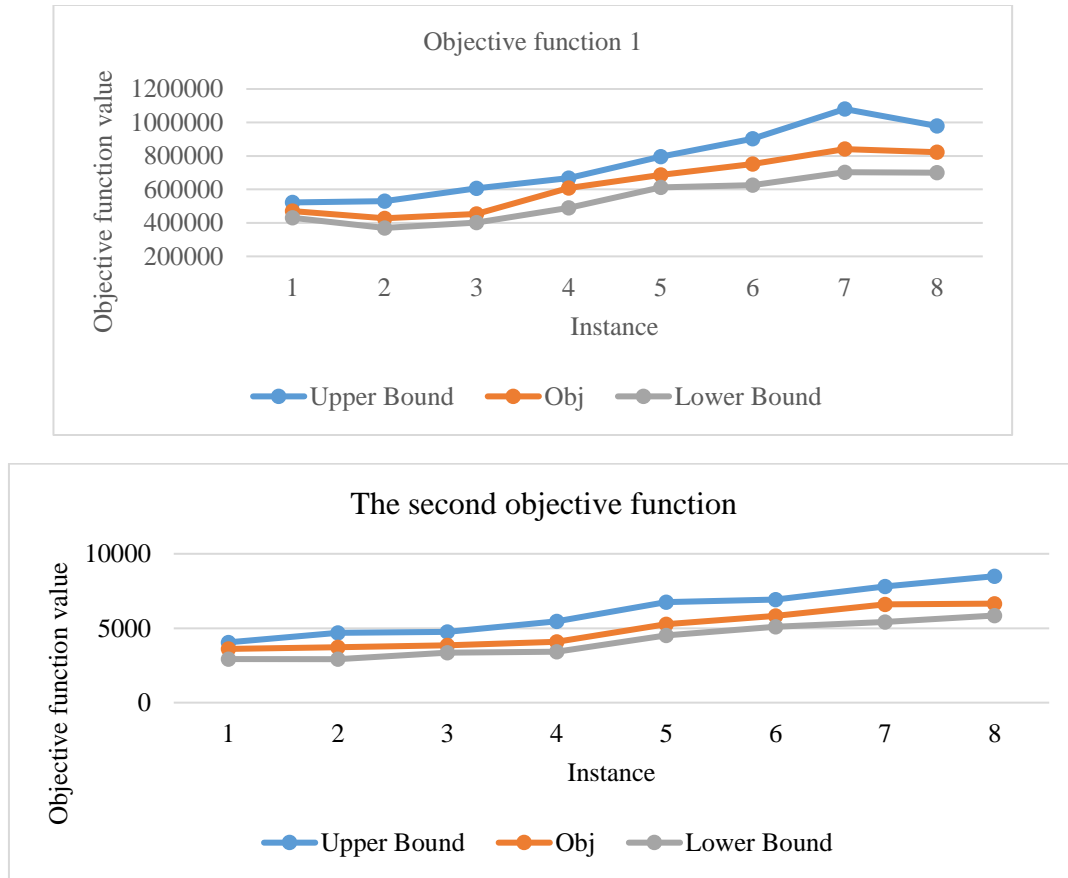


Figure 1. Changes in the first and the second objective functions over eight instances.

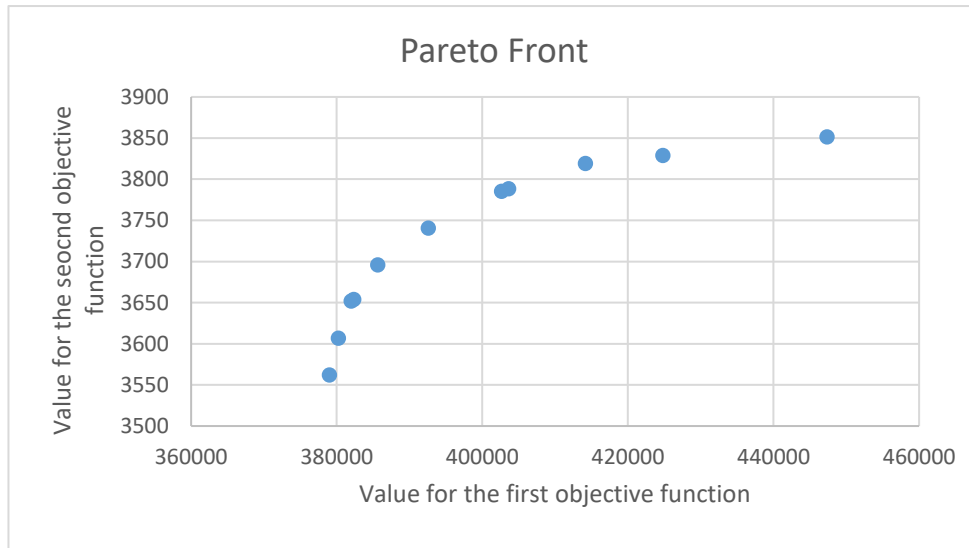
## 5.2. Solving model II by using the $\epsilon$ -constraint method

As a widely applied method in solving the multi-objective problems, this section applies the  $\epsilon$ -constraint method to solve Model II. The  $\epsilon$ -constraint method is the inductive method. In this method, effective answers are calculated and then be involved in the decision to select the preferred answer. When multi-objective model is solved by this method, the user may decide on the basis of the Pareto frontier. Then, the Pareto frontier for at least nine solutions will be shown. Again we solved the example by using the CPLEX. These values are shown in Table 10.

**Table 10.** The computational results associated with the first and second objective functions and for non-dominated points.

Solution	The first objective function value	The second objective function value
1	402683	3785.36
2	382350	3654.11
3	392591	3740.65
4	385674	3695.94
5	403613	3788.36
6	381994	3651.80
7	414169	3819.27
8	380266	3606.88
9	424799	3829.00
10	379003	3562.12
11	447364	3851.62

Figure 2 shows the Pareto frontier for these solutions, in which the horizontal axis represents the values of the first objective function (minimizing total costs), and the vertical axis represents the values of the second objective function (maximizing the purchase score of suppliers).



**Figure 2.** Pareto frontier for the solutions.

As it can be seen in Figure 3 the Pareto frontier needs 10 grid points. The first and the last points of the Pareto frontier present the extreme situations. For instance, in this example the point with the minimum cost and minimum purchase score for the supplier means that in the opinion of supplier, the purchase score is not very important, and only the costs are important. Moreover, the point with the maximum cost and maximum purchase score represents the condition that the decision

maker did not face any cost limitation, and only would like to maximize the purchase score. The middle points of the Pareto frontier shows a balance between two objective functions, as well as the decision makers' priorities should they select a point. Figure 4 shows the normalized Pareto frontier by using Equation (43).

$$d_i = \frac{x_i - x_{min}}{x_{max} - x_{min}}, i = 1, 2, \dots, 10 \quad (43)$$

It worth emphasizing that the only reason for normalizing points of Figure 3 is to have an improved visualization, which helps to perform the analysis better.

### 5.3. Sensitivity Analysis

As part of the model validation we perform a sensitivity analysis for both scenarios of increasing and decreasing the value of demand. Each scenario includes the change in the Pareto frontier, the results and the normalized Pareto frontier, as well as comparison with the primary situation. Note that with a reasonable and logical increase in the value of demand, the values of the first objective function increases (total costs), and the value of the second objective function decreases (service level).

#### Scenario 1. A decrease in the value of demand

Assume that the value of demand has been decreased by 50%. We re-solve the model with the inclusion of new level for demand, and obtain a new Pareto frontier, which is shown in Figure 5. Indeed, Figure 4 compares the points from Scenario 1 with the corresponding points resulted from the main problem (the blue points represent Scenario 1 and the red points represent the main problem).

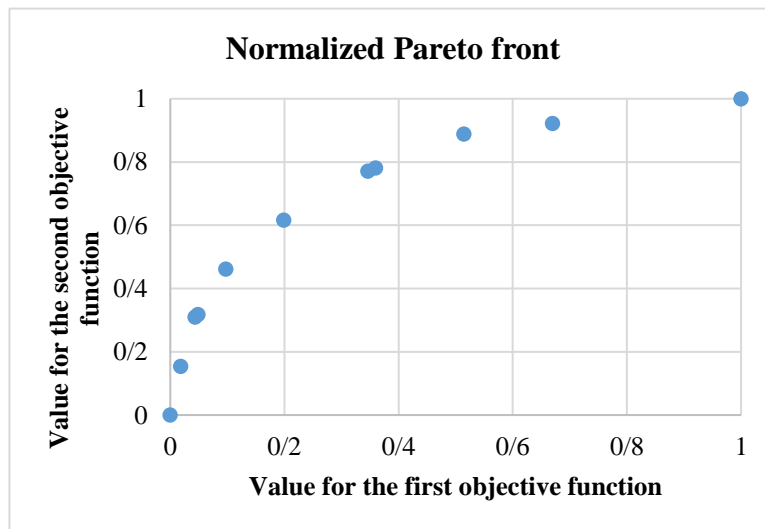


Figure 3. Normalized Pareto frontier for 11 solutions of the example.



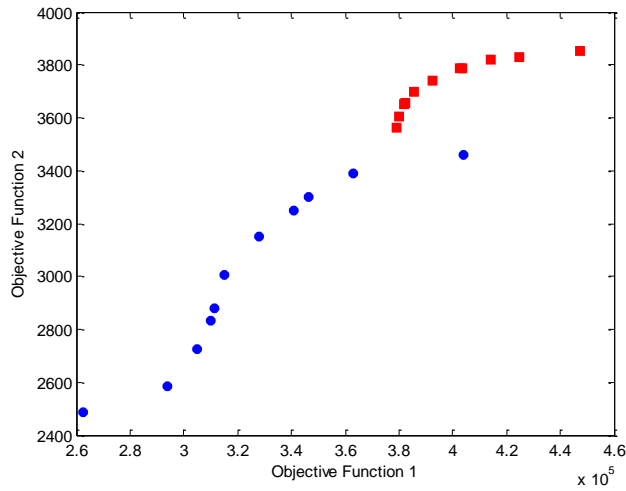


Figure 4. Pareto frontier for Scenario 1 of the example.

**Scenario 2. An increase in the value of demand**

Similarly, let us assume that the value of demand has been increased by 50% (Scenario 2). We re-solve the model with the inclusion of new increased level for demand, and obtain a new Pareto frontier, which is shown in Figure 5.

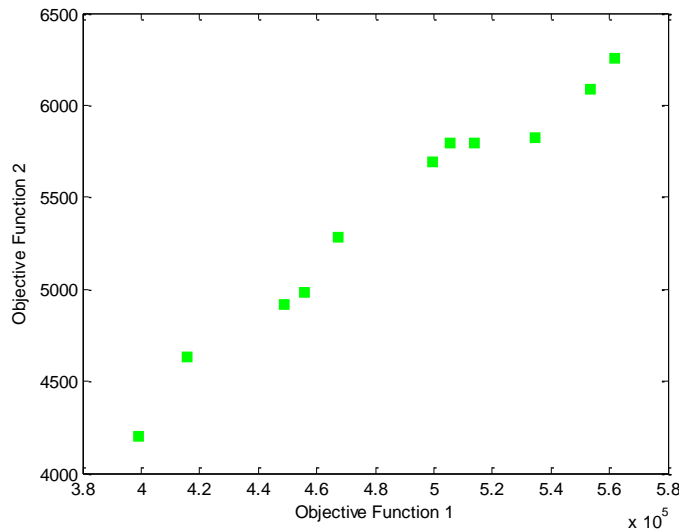
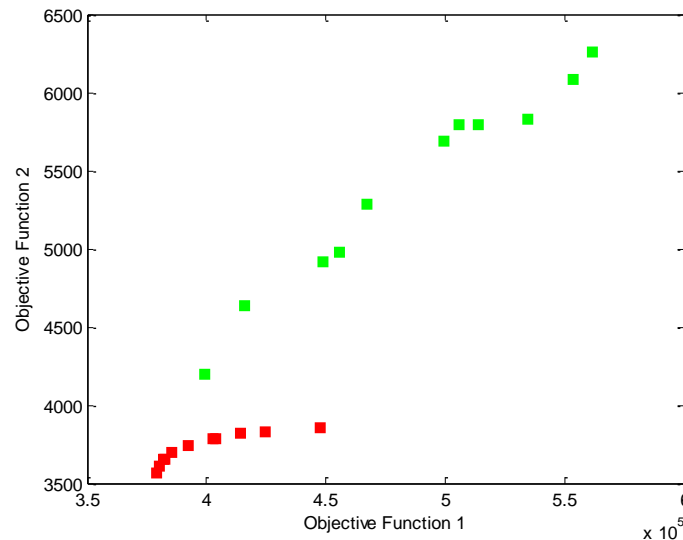


Figure 5. Pareto frontier for Scenario 2 of the example.

Normalized Pareto frontier is shown in Figure 6. Figure 6 compares the points from Scenario 2 with the main problem (the green points show Scenario 2 and the red points show the main

problem). As expected, a decrease in the value of demand shifts the points of the Pareto frontier to the top right. At the end, the points from the Pareto frontier of the main problem (the red points), Scenario 1 (the blue points) and Scenario 2 (the green points) have been illustrated together in one graph (Figure 6).



**Figure 6.** The normalized Pareto frontier associated with Scenario 2, and comparing this with the primary solution

## 5. Conclusion

An integrated approach for supplier selection in the supply chain and order policy from each of them was investigated in this study. In order to achieve the goals of the research both multi-criteria techniques, to select the suppliers (a strategic decision), and optimization methods, determine the optimal order level from each supplier and optimal routing (an operational decision) have been applied. We applied two techniques of “weighting” and “ $\epsilon$ -constraint” methods to solve the optimization model. In addition, the sensibility analysis was conducted to investigate the changes in the problem’s parameters and the environmental conditions.

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