FPGA-based McEliece Cryptosystem using Non-linear Convolutional Codes

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Abstract: The paper reports development of a new version of the McEliece cryptosystem using non-linear convolutional codes. Cascaded convolutional codes are used to be part of the public key with each stage of the cascade separated by a product cipher to increase the security level. Cryptanalysis of the new version of the McEliece cryptosystem is performed using existing attacks of the classical cryptosystem to demonstrate the difficulties in breaking the new cryptosystem. It is shown that, security levels comparable to the original McEliece cryptosystem could be obtained by using smaller public key sizes of the new version if multiple stages of the generator matrix are employed. This aspect makes the new version of the McEliece cryptosystem attractive in mobile wireless networks since it could be ported onto a single Field Programmable Gate Array (FPGA).

1 INTRODUCTION

The McEliece public key cryptography (PKC) has received a lot of attention since it is one of the most attractive options for post-quantum PKC and due to fewer encryption/ decryption operations compared to other PKC schemes such as RSA, ECC and ElGamal. However, low encryption rate and large key size due to Goppa block codes have motivated major research efforts targeting the implementation of the McEliece cryptosystem using alternative codes. Of recent, convolutional codes have been used as an alternative in the implementation of the McEliece cryptosystem (Almeida et al, 2013, 2016 ; Almeida & Napp, 2018; Moufek & Guenda, 2018; Rosenthal & Smarandache, 1999) . Convolutional codes consider the information as a whole sequence and better resist decoding attacks compared to block codes due to the dynamic nature of the cryptosystem. The output sequence depends on the current state of the generator matrix, the transition functions as well as the previous and present input bits. The major challenge with convolutional coding remains an efficient decoding method especially with higher order codes. Decoding methods such as the list decoding (Zigangirov & Osthoff, 1993), threshold decoding algorithm (Massey, 1963) and Information set decoding (Peters, 2010) have been developed. The most prominent decoding method is the Viterbi algorithm. The algorithm has the drawback of an exponential increase in complexity for higher order codes. Hence, an efficient convolutional cryptosystem implementation should involve lower order codes. In (Kumari & Saini, 2016), the authors presented an efficient method of generating polynomials for lower order codes using MATLAB suitable for the Viterbi decoding algorithm.

Existing McEliece cryptosystem implementation using convolutional codes laid emphasis on basic ideas and right parameters of convolutional codes which could be efficient in curbing the attacks of a classical McEliece cryptosystem (Almeida & Napp, 2018). Aspects such as key size and encryption/ decryption complexity were not addressed.

In this paper, we present the implementation of a new variant of the McEliece cryptosystem using non-linear convolutional codes. The number of operations required to effectively carry out a structural attack or an information set decoding attack on the new variant of the McEliece cryptosystem will be used to establish bounds for the key size. The generator matrix, G is like the existing convolutional codes used in the McEliece cryptosystem, the difference in this new method is that, the generator matrices are implemented in stages interspaced with product ciphers. The complexity to decode the ciphertext increases with the number of stages. Cryptanalysis of the new version of the McEliece cryptosystem is performed using existing attacks of the classical cryptosystem to demonstrate the difficulties in

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breaking the new cryptosystem. In addition, it is shown that, security levels comparable to the original McEliece cryptosystem could be obtained by using smaller public key sizes of the new version if multiple stages of the same generator matrix are employed. The aspect of small key sizes makes the new version of the McEliece cryptosystem attractive in mobile wireless networks since it could be ported into a single FPGA. Future research will involve the application of this novel version of the McEliece cryptosystem to wireless cooperative networks.

The complete outline of the paper is as follows. In the next section, a review of non-linear convolutional cryptosystem is presented (Sone, 2015). The non-linear convolutional coding is implemented by inserting product ciphers between conventional convolutional coding blocks. The classical McEliece PKC and the new variant of the McEliece PKC based on the non-linear convolutional codes are presented in section 3. Section 4 presents the cryptanalysis of the novel McEliece cryptosystem. The cryptanalysis is based on assessing the security attacks and the key size required to curb the attacks. Results and discussion are presented in section 5. The section presents a comparative study of the traditional and new variant of the McEliece cryptosystems with respect to the key size. Section 6 presents the FPGA implementation of the new variant of the McEliece cryptosystem. An overview of wireless networks security attacks is presented in section 7. Finally, the conclusion and future work are presented in section 8.

2 NON-LINEAR CONVOLUTIONAL CRYPTOSYSTEM

This section reviews the basics of non-linear convolutional code and the implementation of the non-linear convolutional encoding/ decoding using the basic code and product ciphers.

2.1 Non-linear Convolutional Code Basics

A convolutional code is generated by passing the information sequence to be transmitted through a linear finite-state shift register. The shift register consists of m (k-bit) stages and n linear algebraic function generators. The n linear algebraic function generators produce the n output bits for each k-bit input sequence. Such an encoder produces an (n, k, m) convolutional code. The function generators are assembled into a generator matrix (Lathi, 1998). The generator matrix is specified functionally by using a set of n vectors, with one vector for each of the n modulo-2 adders. Each vector has km dimensions and contains the connections of the encoder to the modulo-2 adder. For illustrative purposes, a (2, 2, 2) convolutional code will be used.

The corresponding (2, 2, 2) convolutional encoder is represented graphically in figure 1.

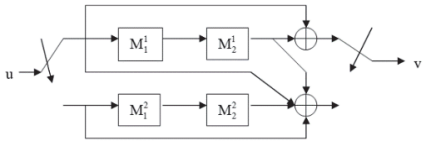


Figure 1: (2, 2, 2) convolutional encoder.

The elements of the generator matrices of the (2, 2, 2) convolutional code are given as

$$g_{m,0} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \; g_{m,1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; g_{m,2} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad (1)$$

The generator matrix, g_m can be obtained from the matrices in (1) and the different states as

$$g_m = \begin{bmatrix} g^1_{m,0} & g^2_{m,1} & g^3_{m,2} \\ 0 & g^2_{m,0} & g^3_{m,1} \\ 0 & 0 & g^3_{m,0} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Coupling of the linear structure in figure 1 onto itself leads to a cascaded non-linear structure. Meta S-boxes (or meta substitution) and permutation set are used to link one linear transducer stage to the next in the cascade (Sone, 2015). Let the S-box and permutation set be chosen as shown in table 1.

Table 1: (a) S-box and (b) Permutation set.

Input	0 0	0 1	1 0	1 1
Output	0 0	1 1	1 0	0 1

(a)

Input	1	2
Output	2	1

(b)

Assuming the initial state for each of the linear transducer stage to be the first state as shown in figure 1, the initial structure of the cascade using the S-box and permutation set is as shown in figure 2.

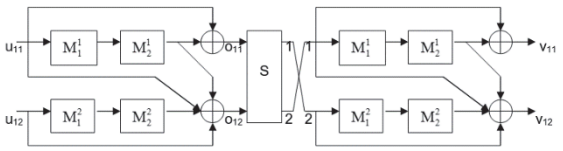


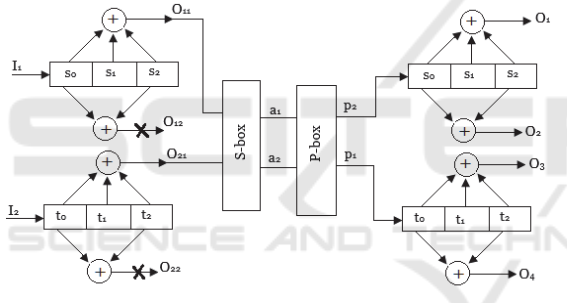
Figure 2: Initial structure of the cascade.

In figure 2, '1' and '2' at the outputs of the S-box, indicates the interconnections due to the permutation set. It is seen in table 1 that, the '1' output from the S-box is connected to the '2' input of the next stage. The same applies to the '2' output which is connected to the '1' input of the next stage.

2.2 Encoding/Decoding using Non-linear Convolutional Code

The constraint length, L for a (n,k,m) convolutional code is given as L = k(m-1). The constraint length is very essential in convolutional encoding since a Trellis diagram which gives the best encoding representation populates after L bits. Hence to encode blocks of n bits, each block must be terminated by L zeros (0s) before encoding.

For illustrative purposes, a non-linear (4,2,3) convolutional code will be used to demonstrate encoding and Viterbi decoding. A possible non-linear (4,2,3) convolutional code showing mod-2 connect-ions and the product cipher is shown in figure 3.



S-box				
$O_{11}O_{21}$	00	01	10	11
a_1a_2	10	11	00	01

P-box		
Input	a_1	a_2
Output	p_2	p_1

Figure 3: 2-stage non-linear (4,2,3) convolutional code.

Example: Encode/ Decode the Message M = 10011

- Encoding process

The constraint length, L = k(m-1) = 2(3-1)= 4

Hence 4 zeros will be appended to message M before encoding. The modified message becomes M′ = 10110000

Transition tables for the (4,2,3) convolutional code shown in the appendix are used to encode the modified message.

- Using the transition tables, the transmitted sequence from the 1st stage is given as T_{in} = 10 01 01 11
- S-box output is given as S = 00 11 11 01
- P-box output is given as P = 00 11 11 10
- Transmitted sequence into the 2nd stage is given as P = 00 11 11 10
- Using the transition tables, the final transmitted sequence which is the output bits from the 2nd stage is given as T_{out} = 0000 1111 0101 1001

- Viterbi decoding process

In performing the Viterbi algorithm, a bit in the sequence T_{out} will be altered. Let the received sequence be T_R = 1000 1111 0101 1001 instead of T_{out} = 0000 1111 0101 1001. The Viterbi algorithm applied to the 2nd stage is summarized in table 2.

Table 2: Viterbi algorithm applied to 2nd stage of (4,2,3) code.

Incoming bits	Input State	Output bits	Output state	Present metric	Cumulative metric
1000	0000	0000	0000	3	3
	0000	0011	0010	1	1
	0000	1100	1000	3	3
	0000	1111	1010	1	1
1111	0000	0000	0000	0	3
	0000	0011	0010	2	5
	0000	1100	1000	2	5
	0000	1111	1010	4	7
	1000	1000	0100	1	4
	1000	1011	0110	3	6
	1000	0100	1100	1	4
	1000	0111	1110	3	6
0101	1010	1010	0101	0	7
	1010	1001	0111	2	9
	1010	0110	1101	2	9
	1010	0101	1111	4	11
1001	1111	0101	0101	2	13
	1111	0110	0111	0	11
	1111	1001	1101	4	15
	1111	1010	1111	2	13

The bits above the arrows will constitute the retrieved sequence from the 2nd stage. Hence, the retrieved sequence is given as, R_1 = 00 11 11 10. This sequence is fed to the P-box.

- P-box output is given as P1 = 00 11 11 01. Sequence, P1 is fed to the S-box
- S-box output is given as S1 = 10 01 01 11

Sequence, S1 is fed into the 1st stage to retrieve the final correct message. The Viterbi algorithm applied to the 1st stage is summarized in table 3.

Table 3: Viterbi algorithm applied to 1st stage of (4,2,3) code.

Incoming bits	Input State	Output bits	Output state	Present metric	Cumulative metric
10	0000	00	0000	1	1
	0000	01	0010	0	0
	0000	10	1000	2	2
	0000	11	1010	1	1
01	1000	10	0100	0	2
	1000	11	0110	1	2
	1000	00	1100	1	3
	1000	01	1110	2	4
01	1110	01	0101	2	6
	1110	00	0111	1	5
	1110	11	1101	1	5
	1110	10	1111	0	4
11	0101	11	0000	2	8
	0101	10	0010	1	7
	0101	01	1000	1	7
	0101	00	1010	0	6

For a good trellis, the final state is the 'all zero' state as seen in the winning path in table 3. The final received sequence is identical to the original transmitted message of M′ = R_{final} = 10110000 despite the first bit error. Hence, using the non-linear convolutional code, the error bit was identified and corrected.

3 A NEW VARIANT OF THE MCELIECE CRYPTOSYSTEM

This section explains the implementation of the new variant of the McEliece cryptosystem using the non-linear convolutional code in combination with a scrambled invertible matrix and a permutation matrix.

3.1 The Classical McEliece Cryptosystem

- Key generation
  - ❖ Pick a random [n, k, 2t + 1] linear code, C where n is the number of bits for codeword; k is the number of message bits an t is the number of errors the code can correct
  - ❖ Compute a k x n generator matrix, G for C
  - ❖ Generate a random k x k binary non-singular (invertible) matrix S
  - ❖ Generate a random n x n permutation matrix P
  - ❖ Compute k x n matrix G′ = SGP
  - ➢ Public key is (G′, t)
  - ➢ Private key is (S, G, P, D) where D is the efficient decoding algorithm
- Encryption
  - ❖ Message, $m \in \{0, 1\}^k$
  - ❖ Random vector, $e \in \{0, 1\}^n$
  - ❖ Ciphertext, $c = mG' + e$
- Decryption
  - ❖ Ciphertext, $c \in \{0, 1\}^n$
  - ❖ Compute $CP^{-1} = (mS)G + eP^{-1}$
    Since (mS)G is a valid codeword for the chosen linear code and eP^{-1} has weight t, the decoding algorithm, D can be applied to CP^{-1} to obtain $c' = mS$
  - ❖ Compute $m = c'S^{-1}$.

The difficulty of decoding a random encoder, known to be an NP hard problem underscores the security of McEliece cryptosystem. This is possible for high order block codes such as 1024-bit code.

3.2 The New Variant of the McEliece Cryptosystem

In the new variant of the McEliece cryptosystem proposed in this research, the key parameters are follows:

- Public key: (G′, t);
- Private key: (S, G, s_{box}, p_{box}, P, D) where s_{box} and p_{box} are the additional keys from the product cipher.

G′ corresponds to a k x n non-linear convolutional code that is permutation-equivalent to the chosen secret key such that P permutes the columns of the non-linear convolutional code, G and S switches to a different basis of the same code.

In section 2, aspects of the private key such as the encoding/ decoding, D; states and transition functions of generator matrix, G; keys for the product cipher s_{box}, p_{box} were presented.

The permutation matrix, P used in this research is matrix $P(D,D^{-1}) \in \mathbb{F}^{nxn}$ developed in (Almeida & Napp, 2018).

Meanwhile, in classical McEliece cryptosystem, $c' = mS$ is synonymous to scrambling data m to obtain c′ where $m = c'S^{-1}$ is equivalent to descrambling. The scrambling method will be used for the implementation of the invertible matrix, S in this research since it involves shift registers that is easy to implement in an FPGA.

A simple scrambler and descrambler in figure 4 will be used to explain the proposed invertible matrix, S (Lathi, 1998).

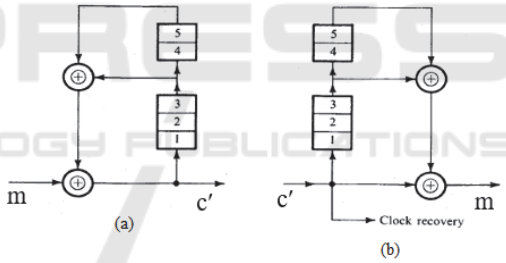


Figure 4: (a) scrambler and (b) descrambler.

The scrambler consists of a feedback shift register and the matching descrambler has a feedforward shift register. If m is the input sequence to the scrambler, then

$$m \oplus D^3 c' \oplus D^5 c' = c'$$

where D represents the delay operator, that is $D^n c'$ is the sequence c′ delayed by n units. Adding $(D^3 \oplus D^5)$ c′ to both sides of the equation gives

$$m = c' \oplus (D^3 \oplus D^5) c' = [1 \oplus (D^3 \oplus D^5)] c' = (1 \oplus F) c'$$

where $F = D^3 \oplus D^5$

To design the descrambler at the receiver, we start with c′ and perform the equation

$$m = c' \oplus F c' = c' \oplus (D^3 \oplus D^5) c'$$

Let message, m = 1010101 be fed into the scrambler.

Initially c′ = m, and the sequence m enters the register and is returned as $(D^3 \oplus D^5)m = Fm$ through the feedback path. This new sequence Fm enters the register and is returned as F^2m, and so on. Hence

$$c' = m \oplus Fm \oplus F^2m \oplus F^3m \oplus \ldots$$

Recognizing that

$$F = D^3 \oplus D^5$$

we have

$F^2 = (D^3 \oplus D^5)(D^3 \oplus D^5) = D^6 \oplus D^{10} \oplus D^8 \oplus D^8 = D^6 \oplus D^{10}$

Since $D^8 \oplus D^8 = 0$

Similarly,

$F^3 = (D^6 \oplus D^{10})(D^3 \oplus D^5) = D^9 \oplus D^{11} \oplus D^{13} \oplus D^{15}$

and so on.

Hence

$c' = (1 \oplus D^3 \oplus D^5 \oplus D^6 \oplus D^9 \oplus D^{10} \oplus D^{11} \oplus D^{13} \oplus D^{15} \oplus \ldots)m$

Because $D^n m$ is simply the sequence m delayed by n bits, various terms in the preceding equation correspond to the following sequences:

m = 1 0 1 0 1 0 1
D^3m = 0 0 0 1 0 1 0 1 0 1
D^5m = 0 0 0 0 0 1 0 1 0 1 0 1
D^6m = 0 0 0 0 0 0 1 0 1 0 1 0 1
D^9m = 0 0 0 0 0 0 0 0 0 1 0 1 0 1
c′ = 1 0 1 1 1 0 0

It is worth noting that, the string c′ is calculated vertically using mod-2 arithmetic and input sequence, m has 7 digits hence only 7 digits of the scrambler output are retained.

When sequence c′ is applied to the input of the descrambler, the output is the original sequence, m

$m = (1 \oplus D^3 \oplus D^5)\, c'$
c′ = 1 0 1 1 1 0 0
D^3c' = 0 0 0 1 0 1 1 1 0 0
D^5c' = 0 0 0 0 0 1 0 1 1 1 0 0

mod-2 arithmetic gives the 7-bit sequence 1010101 which is identical to the input sequence m = 1010101

Based on the afore-mentioned analysis, a k x k invertible matrix, S and the scrambled message c′ could be deduced from a k-bit message fed into the scrambler as follows:

- Each row of the k x k matrix, S contains elements of the shifted message, m deduced from the scrambler
- Each bit of the scrambled message, c′ is computed from the sum of each column of the k x k matrix.

Hence the k x k matrix, S of the scrambler in figure 4 is given as

$$S = \begin{pmatrix} D^0m(0) & D^0m(1) & D^0m(2) & D^0m(3) & D^0m(4) & D^0m(5) & D^0m(6) & D^0m(7) & D^0m(k) \\ D^3m(0) & D^3m(1) & D^3m(2) & D^3m(3) & D^3m(4) & D^3m(5) & D^3m(6) & D^3m(7) & D^3m(k) \\ D^5m(0) & D^5m(1) & D^5m(2) & D^5m(3) & D^5m(4) & D^5m(5) & D^5m(6) & D^5m(7) & D^5m(k) \\ D^6m(0) & D^6m(1) & D^6m(2) & D^6m(3) & D^6m(4) & D^6m(5) & D^6m(6) & D^6m(7) & D^6m(k) \\ D^9m(0) & D^9m(1) & D^9m(2) & D^9m(3) & D^9m(4) & D^9m(5) & D^9m(6) & D^9m(7) & D^9m(k) \\ D^{10}m(0) & D^{10}m(1) & D^{10}m(2) & D^{10}m(3) & D^{10}m(4) & D^{10}m(5) & D^{10}m(6) & D^{10}m(7) & D^{10}m(k) \\ D^{11}m(0) & D^{11}m(1) & D^{11}m(2) & D^{11}m(3) & D^{11}m(4) & D^{11}m(5) & D^{11}m(6) & D^{11}m(7) & D^{11}m(k) \\ D^{13}m(0) & D^{13}m(1) & D^{13}m(2) & D^{13}m(3) & D^{13}m(4) & D^{14}m(5) & D^{14}m(6) & D^{13}m(7) & D^{13}m(k) \end{pmatrix} \quad (3)$$

Similarly, for the descrambler, the matrix S^{-1} and the original message, m are obtained as follows:

- Each row of the matrix, S^{-1} contains elements of the shifted scrambled message, c′ deduced from the descrambler
- Each bit of the message, m is computed from the sum of each column of the matrix, S^{-1}

Hence the matrix, S^{-1} of the descrambler in figure 4 is given as

$$S^{-1} = \begin{pmatrix} D^0c'(0) & D^0c'(1) & D^0c'(2) & D^0c'(3) & D^0c'(4) & D^0c'(5) & D^0c'(6) & D^0c'(7) & \ldots & D^0c'(k) \\ D^3c'(0) & D^3c'(1) & D^3c'(2) & D^3c'(3) & D^3c'(4) & D^3c'(5) & D^3c'(6) & D^3c'(7) & \ldots & D^3c'(k) \\ D^5c'(0) & D^5c'(1) & D^5c'(2) & D^5c'(3) & D^5c'(4) & D^5c'(5) & D^5c'(6) & D^5c'(7) & \ldots & D^5c'(k) \end{pmatrix} \quad (4)$$

4 CRYPTANALYSIS

The security of the cryptosystem is based on two computationally hard problems namely, exhaustive search of the key space and maximum-likelihood decoding (syndrome decoding). Therefore, the two types of attacks, which are principally structural and decoding, will be the basis of the cryptanalysis of the new variant of the McEliece cryptosystem. In this section, cryptanalysis will explore the additional security due to the non-linear convolutional cryptosystem and not the entire new variant which involves also the invertible matrix, S and the permutation matrix, P. Hence, the cryptanalysis will establish baseline values for the key sizes for the new variant of the McEliece cryptosystem.

4.1 Structural Attack

Structural attacks against the McEliece cryptosystem involve recovering of the secret key from the public key, G′ in order to determine an equivalent code from c generated by G. It is worth noting that, in classical convolutional codes, G is a generator matrix

characterised by the states of the mod-2 adders and the transition functions, whereas in the new variant of the McEliece cryptosystem, G is a generator matrix that is coupled to the product cipher. Hence, in addition to the states and transition functions of the generator matrix, the combinations in the S-boxes and P-boxes must be considered.

Therefore, a successful structural attack on the new variant of the McEliece cryptosystem should require a minimum number of plaintext – ciphertext pairs in order to reveal the following parameters:

- States and transition functions of the generator matrix;
- Bit shuffling and permutation combinations of the S-box and P-box.

4.1.1 States and Transition Functions

Generally, a (k, k, m) convolutional transducer with q states and having p blocks of input vector, u, will have the generator matrix given as (Biggs, 2008)

$$g_{kp}(u) = \begin{bmatrix} g_0^0 & g_1^1 & \cdots & g_m^m & & & \\ & g_0^1 & \cdots & g_{m-1}^m & g_m^{m+1} & & \\ & & \ddots & & & \ddots & \\ & & & \ddots & g_0^{p-1} & \cdots & g_m^{m+p-1} \end{bmatrix} \quad (5)$$

Gaussian elimination attack could be effectively mounted on the generator matrix to reveal all the states of the cryptosystem (Biggs, 2008). The attack is based on forming simultaneous equations using a (k, k, m) convolutional code function, t given by

$$t(u) = u * g_{ixj} \quad (6)$$

where u is a row vector of message bits and g_{ixj} is a generator matrix. To illustrate the principle involved in this attack, the (2, 2, 2) cryptosystem in section 2.1 with three states will be used. This simple cryptosystem has a readable generator matrix and the analysis will be extended to a general case of a (k, k, m) cryptosystem with q states.

Consider the (2, 2, 2) cryptosystem with three states as follows:

$$g_0^0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad g_1^0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad g_2^0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$g_0^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad g_1^1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad g_2^1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$g_0^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad g_1^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad g_2^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

and a proposed transition function, f given in table 4.

Table 4: Transition table for (2, 2, 2) cryptosystem.

f	[0 0]	[0 1]	[1 0]	[1 1]
1	2	3	3	1
2	3	1	2	3
3	3	2	1	2

With respect to the (2, 2, 2) cryptosystem, using p = 5 blocks of input, the generation matrix becomes

$$g_{10}(u) = \begin{bmatrix} g_0^0 & g_1^1 & g_2^2 & & \\ & g_0^1 & g_1^2 & g_2^3 & \\ & & g_0^2 & g_1^3 & g_2^4 \\ & & & g_0^3 & g_1^4 \\ & & & & g_0^4 \end{bmatrix} \quad (7)$$

The intruder will have to determine 48 unknown matrix elements in the matrix $g'_{10}(u)$. The unknown generator matrix is given as

$$g'_{10}(u) = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} & x_{1,5} & x_{1,6} & & & & & \\ x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} & x_{2,5} & x_{2,6} & & & & & \\ & & x_{3,3} & x_{3,4} & x_{3,5} & x_{3,6} & x_{3,7} & x_{3,8} & & & \\ & & x_{4,3} & x_{4,4} & x_{4,5} & x_{4,6} & x_{4,7} & x_{4,8} & & & \\ & & & & x_{5,5} & x_{5,6} & x_{5,7} & x_{5,8} & x_{5,9} & x_{5,10} \\ & & & & x_{6,5} & x_{6,6} & x_{6,7} & x_{6,8} & x_{6,9} & x_{6,10} \\ & & & & & & x_{7,7} & x_{7,8} & x_{7,9} & x_{7,10} \\ & & & & & & x_{8,7} & x_{8,8} & x_{8,9} & x_{8,10} \\ & & & & & & & & x_{9,9} & x_{9,10} \\ & & & & & & & & x_{10,9} & x_{10,10} \end{bmatrix} \quad (8)$$

Let $(u_1, v_1) = \left([0100111100], [0100011001]\right)$ be plaintext – ciphertext pair equivalent to 5 blocks of 2-bit input. By applying Gaussian elimination attack (Sone, 2015) to the unknown matrix in (7) the intruder obtains the following matrix

$$g'_{10}(u) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & & & & & \\ 0 & 1 & 0 & 0 & 1 & 1 & & & & & \\ & & 1 & 1 & 0 & 0 & 1 & 1 & & & \\ & & 0 & 1 & 0 & 0 & 1 & 1 & & & \\ & & & & 1 & 1 & 0 & 0 & 1 & 0 & \\ & & & & 0 & 1 & 0 & 0 & 1 & 1 & \\ & & & & & & 1 & 1 & 0 & 0 & \\ & & & & & & 0 & 1 & 0 & 0 & \\ & & & & & & & & 1 & 1 & \\ & & & & & & & & 0 & 1 & \end{bmatrix} \quad (9)$$

Close examination of $g'_{10}(u)$ in (9) shows that $g'_{10}(u)$ contains two distinct states. One of the states is given by columns 5 and 6 (or equivalently by columns 9 and 10) and the other state is given by columns 7 and 8. The third state could be found by using another plaintext-ciphertext pair. Let (u_2, v_2) =

$([1101010100], [10\ 01001001])$ be a second plaintext-ciphertext pair used by the attacker. Applying Gaussian elimination attack, the unknown matrix becomes

$$g'_{10}(u) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & & & & \\ 0 & 1 & 0 & 0 & 1 & 1 & & & & \\ & & 1 & 1 & 0 & 0 & 1 & 1 & & \\ & & 0 & 1 & 0 & 0 & 1 & 1 & & \\ & & & & 1 & 1 & 0 & 0 & 1 & 1 \\ & & & & 0 & 1 & 0 & 0 & 0 & 1 \\ & & & & & & 1 & 1 & 0 & 0 \\ & & & & & & 0 & 1 & 0 & 0 \\ & & & & & & & & 1 & 1 \\ & & & & & & & & 0 & 1 \end{bmatrix} \quad (10)$$

$g'_{10}(u)$ in (10) reveals the last state of the cryptosystem in columns 9 and 10, namely

$$\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right)$$

Thus, by using two plaintext-ciphertext pairs, the states of the cryptosystem are found, and the cryptosystem is partially broken since the remaining private keys, namely the transition functions and keys used in product cipher also need to be unveiled.

Let the state

$$\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right)$$

be denoted as i_1, the state

$$\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)$$

be denoted as i_2 and the state

$$\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right)$$

as i_3.

In a convolutional cryptosystem, the private parameter f compares the present state and the input in order to output the next state of the cryptosystem. In the generator matrix, $g'_{10}(u)$ in (9) the transitions are $i_1 \rightarrow i_2 \rightarrow i_1$ and the input vector has $u_1\,[5, 6] = [1\ 1]$. Using (7), $u_1\, g'_{10}(u_1) = v_1$, this implies

$$f_1\,(i_1, u_1\,[5, 6]) = f_1\,(i_1, [1\ 1]) = i_2 \quad (11)$$

where $i_2 = g'_{10}[7, 8]$. Also

$$f_2\,(i_2, u_1\,[7, 8]) = f_2\,(i_2, [1\ 1]) = i_1 \quad (12)$$

where $i_1 = g'_{10}[9, 10]$

Similarly, the transition of $g'_{10}(u)$ in (8) are $i_1 \rightarrow i_2 \rightarrow i_3$ and $u_2\, g'_{10}(u_2) = v_2$.

Using $u_2\,[5, 6] = [0\ 1]$, this will generate columns 7 and 8 from columns 5 and 6 in the generator matrix $g'_{10}(u)$, that is

$$f_3\,(i_1, [0\ 1]) = i_2 \quad (13)$$

and

$$f_4\,(i_2, [0\ 1]) = i_3 \quad (14)$$

Hence using ten plaintext – ciphertext pairs, all the states of the cryptosystem are revealed and four functions f_1, f_2, f_3, f_4 out of the twelve values of the private parameter f are revealed. However, by using additional twenty plaintext-ciphertext pairs, the remaining eight values of the private parameter f are revealed. Hence at least thirty plaintext-ciphertext pairs are required for all values of parameter f and states to be revealed.

Generally, for a (k, k, m) cryptosystem with q states and p blocks of message bits, each matrix $g_{kp}(u)$ reveals at most p – k – 1 values of f. Since there will be $(q \cdot 2^k)$ values of f, where q is the number of states, the minimum number of plaintext-ciphertext pairs (u, v) required to reveal all values of private parameter f and the states is given as

$$N_1 = \frac{q \cdot 2^k}{p - k - 1} \quad (15)$$

Hence, total number of plaintext-ciphertext pairs required,

$$N_{T1} = kpN_1 \quad (16)$$

For the (2,2,2) convolutional code

$N_1 = 3x2^2/ (5 – 2- 1) = 3$ and $N_{T1} = 2x5x3 = 30$ pairs

Therefore, for a successful structural attack for a classical convolutional code, G the total number of representation code which the attacker has to compare to ciphertext c′ generated by G′ has to be N_{T1} for an (n,k,m) code with q states.

4.1.2 Shuffling Boxes and Permutation Sets

To analyse all the permutations, the minimum number of plaintext – ciphertext pairs required for an (n,k,m) code with p blocks of k-bit input is

$$N_2 = pk! \quad (17)$$

In addition, to analyse all the different 2-bit combinations in the s-boxes, the minimum number of plaintext – ciphertext pairs is given as

$$N_3 = 2^{2k} \quad (18)$$

Hence, for a successful structural attack for either

- one stage or
- multiple stages with the same generator matrix and product cipher

of the new non-linear convolutional code, G the total number of representation code which the attacker must compare to ciphertext c′ generated by G′ has to be N_T for an (n,k,m) code with q states and is given as

$$N_T = pk! \,.\, 2^{2k} \,.\, \frac{q.2^k}{p-k-1} \tag{19}$$

For μ stages of the new variant of the McEliece cryptosystem using stages with different configuration, the total number of representation code which the attacker has to compare to ciphertext c′ generated by G′ has to be N_{Total} for an (n,k,m) code with q states and is given as

$$N_{Total} = \left[pk! \,.\, 2^{2k} \,.\, \frac{q.2^k}{p-k-1}\right]^{\mu} \tag{20}$$

4.2 Decoding Attack

A decoding attack consists of decoding the intercepted ciphertext. The cost of the attack depends on the parameters of c′ namely, length, dimension and error-correcting capability since the underlying code and c′ are equivalent.

If a message of length n bits is received, then the possible number of codewords are 2^n.

For an (n,k,m) convolutional code, only 2^{kL} codewords are valid of the possible 2^n. The Viterbi algorithm applies the maximum-likelihood principles to limit the comparison to 2^{kL} surviving paths instead of checking all the paths where L = constraint length = k(m-1) .

For μ stages of the new variant of the McEliece cryptosystem using non-linear convolutional codes, the total number of operations the attacker must perform in order to decode the ciphertext has to be N_{Tot} for an (n,k,m) code with q states and is given as

$$N_{Tot} = [pk! \,.\, 2^{2k} \,.\, 2^{kL}]^{\mu} \tag{21}$$

Note that, in establishing (21) the product cipher was used in conjunction with the Viterbi algorithm.

5 RESULTS AND DISCUSSION

There are several ways to attack the McEliece cryptosystem. In this section, we shall analyse the structural and decoding attacks using appropriate convolutional codes and compare the results to the baseline parameters of the original McEliece cryptosystem.

Original parameters n=1024, k =524, t=50, suggested by McEliece in (McEliece, 1978) are now considered insecure, as they only offer approximately 50-bit security (Bernstein et al, 2011). There is no clear consensus on recommended parameters for MECS for various typical security levels. There have been various theoretical articles suggesting and analysing security of MECS parameters (McEliece, 1978; Bernstein et al, 2011; Biswas & Sendrier, 2008; Eisenbarth et al, 2009).

In this research, we shall consider the baseline parameters n=1024, k =524, t=50 as the basis for comparison with the new variant of the McEliece cryptosystem. It is worth noting that, the expressions deduced in section 4 are baseline number of operations required for the structural and decoding attacks since only the non-linear convolutional code, G was considered instead of the entire public key G′ = SGP.

5.1 Appropriate Polynomials

There are many choices for polynomials for any m order code where m is the number of registers for an (n,k,m) code. The polynomials do not all result in output sequences that have good error protection properties. Petersen and Weldonís book contain a complete list of these polynomials (Peterson & Weldon, 1972) and good polynomials are found from this list usually by computer simulation/MATLAB (Arasteh, 2006). There is no known constructive way for selection of generator polynomials, however a convolutional code can be analysed to find its distance properties. A good convolutional code has large free hamming distance (Kumari & Saini, 2016).

Existing works on McEliece cryptosystem based on convolutional codes used high order convolutional codes such as (305,150,4), (570,421,3) and (284,71,5) (Moufek & Guenda, 2015). Such high order codes are difficult to implement in hardware and the decoding process is quite cumbersome. Hence, low order codes developed in (Kumari & Saini, 2016) will be adopted.

5.2 Structural Attacks

In the general case, the structural attack quickly becomes infeasible for any reasonable choice of parameters for the Goppa code (Loidreau & Sendrier, 2001). For the original McEliece parameters with codeword, n = 1024, message bits, k = 512 and error-

Table 5: Total number of corresponding representation code, N_{Total}.

(n,k,m) convolutional code	Octal value of generator matrices	Error-correcting capability, t	N_{Total}		
			p=10; μ=15	p=20; μ=10	p=15; μ=10
(3,1,3)	6,5,7	10 errors	2^{367}	2^{637}	2^{429}
	3,4,7				
	7,6,5	20 errors			
	5,7,2				
(3,1,4)	17,15,3	20 errors	2^{382}	2^{647}	2^{439}
	16,13,15				

Table 6: Total number of corresponding representation code, N_{Tot}.

(n,k,m) convolutional code	Octal value of generator matrices	Error-correcting capability, t	N_{Total}		
			p=5; μ=6	p=10; μ=2	p=8; μ=4
(3,1,3)	6,5,7	10 errors	2^{64}	2^{51}	2^{75}
	3,4,7				
	7,6,5	20 errors			
	5,7,2				
(3,1,4)	17,15,3	20 errors	2^{70}	2^{53}	2^{80}
	16,13,15				

correcting capability, t = 50, this mounts up to roughly 2^{461} total number of representation code which the attacker has to compare to ciphertext c′ generated by G′ (Loidreau & Sendrier, 2001). Using low order codes developed in (Kumari & Saini, 2016), (20) is used to compute N_{Total} deduced in section 4.1 and results compared to 2^{461} obtained using the original McEliece cryptosystem. The results for the best generator polynomials in terms of performance for code rate 1/3 are summarized in table 5 for appropriate number of blocks, p, number of states, $q = 2^m$ and number of stages, μ in the non-linear cascaded encoder. Appropriate values for p, q and μ which give N_{Total} comparable to 2^{461} are used.

Hence, using low order codes there is the possibility of attaining high security levels comparable to the original 1024-bit McEliece cryptosystem if appropriate values of the number of states of the non-linear convolutional code and number of blocks for k-bit input are chosen as shown in table 5. As mentioned earlier, the values displayed in table 5 are baseline number of operations required for the structural attack since only the non-linear convolutional code, G was considered instead of the entire public key G′ = SGP. Hence, higher values of N_{Total} compared to those displayed in table 5 could be obtained using G′ for the same values of p, q and μ.

5.3 Decoding Attacks

A decoding attack consists of decoding the intercepted ciphertext. Since the underlying code and c′ are equivalent, they have the same error-correcting capability. Thus, the cost of the attack depends only on the parameters of c′– its length, dimension and error-correcting capability. When the length is n = 1024, the dimension is k = 524 and the error-correcting capability is t = 50, decoding one word requires 2^{64} binary operations (Loidreau, 2000).

Similarly, using low order codes, (21) is used to compute N_{Tot} deduced in section 4.2 and results compared to 2^{64} obtained using the original McEliece cryptosystem. The results are summarized in table 6 for appropriate number of blocks, p, number of states, $q = 2^m$ and number of stages, μ in the non-linear cascaded encoder.

Hence, using low order codes there is the possibility of attaining high security levels with $N_{Tot} \geq 2^{64}$ for the new variant of the McEliece cryptosystem based on the non-linear convolutional code as shown in table 6.

6 FPGA IMPLEMENTATION

A good overview of the existing hardware implementations of the McEliece cryptosystem can be found in (Repka & Cayrel, 2014). Most of the implementations require external memory for both the public and private key structures. In (Shoufan et al, 2009), the authors implemented a MECS with n = 2048, t = 50 in a Virtex 5 FPGA using 84% of slices and 50% of BRAMs (2700 Kb).

The implementation of the new variant of the McEliece cryptosystem is ported onto a Virtex 4 FPGA and the architecture has the following features:

- An instantiated encryption package;
- An instantiated decryption package;
- A finite state machine;
- An instantiated look-up table used in the Viterbi decoding process.

An important aspect in FPGA-based MECS is to obtain synthesizable architectures from the multi-dimensional arrays, that is, arrays with more than one index such as matrices. Multi–dimensional arrays are not allowed for hardware synthesis. One way around this, is to declare two one–dimensional array types. This approach is easier to use and more representative of actual hardware. The VHDL code used in this research to declare the two one–dimensional array types for 256x256 matrix is shown in figure 5.

```
Subtype Depth_Typ is Integer range 0 to 255;
Subtype Width_Typ is Integer range 255 downto 0;
Subtype Data_Typ is Bit_vector (Width_Typ);
Type Memory_Typ is array (Depth_Typ) of Data_Typ;
```

Figure 5: VHDL code for synthesizable 256 x 256 matrix.

The device utilization summary for a (3,1,3) non-linear convolutional code MECS for number of blocks, p = 15 and number of stages, μ = 10 is as follows:

- Number of slices: 66816 out of 89088 75%

- Number of slice Flip flops: 124720 out of 178176 70%

7 WIRELESS NETWORKS SECURITY ATTACKS

Threats that violate the security criteria of wireless networks are generally called security attacks. The main attacks are impersonation, eavesdropping, Denial of Service and Sensing (DoSS), sybil, node capture, selective forwarding, and various routing attacks. Some of these attacks which are related to the confidentiality property of secure communication could be circumvented using the new variant of the McEliece cryptosystem. McEliece based digital signature scheme when implemented using the new variant, could be used to curb other security attacks which are related to authentication of the wireless network nodes. It is widely believed that, code-based cryptosystem do not allow practical digital signatures. However, McEliece based digital signature scheme has been proposed in (Courtois et al, 2001). Future research work will implement a McEliece based digital signature scheme using the new variant in order to curb the above-mentioned attacks in wireless cooperative networks.

8 CONCLUSION

In this paper, a new variant of the McEliece cryptosystem using non-linear convolutional codes is proposed. The rationale in designing the new variant is to establish key sizes which could enable implementation of the McEliece cryptosystem in a single FPGA device with ultimate application in mobile wireless communication. The new variant of the McEliece cryptosystem is implemented using non-linear convolutional codes, a scrambled matrix and a permutation matrix. The non-linear convolutional code is a combination of the conventional convolutional code and product ciphers. The baseline key sizes used in the implementation depends on the number of blocks of the k-bit input, the number of stages in the n-cascaded non-linear convolutional cryptosystem and the number of states in the generator matrices. The results of this research show that, small key sizes used to establish the matrices in the McEliece cryptosystem could be attained for 15 blocks of k-bit input and 10 stages of cascaded convolutional codes to prevent structural attacks and 5 blocks of k-bit input and 6 stages of convolutional codes to prevent decoding attacks. The entire scheme was implemented in a Virtex-4 FPGA to circumvent the key management drawback associated with the additional keys due to the product ciphers.

Future research will focus on two fundamental problem areas that must be addressed if wireless networks such as cooperative networks are to have security comparable to traditional networks. These areas are:

- Trust establishment, key management and membership;
- Network availability and routing security

The new variant of the McEliece cryptosystem will be used to explore these problem areas by implementing efficient encryption and authentication schemes. The implementation will consider the specificities of wireless cooperative networks such as, limited energy; limited memory; transient connectivity and availability; shared physical medium amongst others.

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APPENDIX

Transition Tables

Input Bits (Stage 1) I_1 I_0	Input State (Stage 1) S_3 S_2 S_1 S_0	Output Bits (Stage 1) O_{11} O_{10}	Output State (Stage 1) S_3 S_2 S_1 S_0
0 0	0 0 0 0	0 0	0 0 0 0
0 1	0 0 0 0	0 1	0 0 1 0
1 0	0 0 0 0	1 0	1 0 0 0
1 1	0 0 0 0	1 1	1 0 1 0
0 0	0 0 0 1	0 1	0 0 0 0
0 1	0 0 0 1	0 0	0 0 1 0
1 0	0 0 0 1	1 1	1 0 0 0
1 1	0 0 0 1	1 0	1 0 1 0
0 0	0 0 1 0	0 1	0 0 0 1
0 1	0 0 1 0	0 0	0 0 1 1
1 0	0 0 1 0	1 1	1 0 1 1
1 1	0 0 1 0	1 0	1 0 0 1
0 0	0 0 1 1	0 0	0 0 0 1
0 1	0 0 1 1	0 1	0 0 1 1
1 0	0 0 1 1	1 0	1 0 0 1
1 1	0 0 1 1	1 1	1 0 1 1
0 0	0 1 0 0	1 0	0 0 1 0
0 1	0 1 0 0	1 1	0 0 1 1
1 0	0 1 0 0	0 0	1 0 0 0
1 1	0 1 0 0	0 1	1 0 1 0
0 0	0 1 0 1	1 1	0 0 0 0
0 1	0 1 0 1	1 0	0 0 1 0
1 0	0 1 0 1	0 1	1 0 0 0
1 1	0 1 0 1	0 0	1 0 1 0
0 0	0 1 1 0	1 1	0 0 0 1
0 1	0 1 1 0	1 0	0 0 1 1
1 0	0 1 1 0	0 1	1 0 0 1
1 1	0 1 1 0	0 0	1 0 1 1
0 0	0 1 1 1	1 0	0 0 0 1
0 1	0 1 1 1	1 1	0 0 1 1
1 0	0 1 1 1	0 0	1 0 0 1
1 1	0 1 1 1	0 1	1 0 1 1

Input Bits (stage 1) I_1 I_0	Input State (Stage 1) S_3 S_2 S_1 S_0	Output Bits (Stage 1) O_{11} O_{10}	Output state (Stage 1) S_3 S_2 S_1 S_0
0 0	1 0 0 0	1 0	0 1 0 0
0 1	1 0 0 0	1 1	0 1 1 0
1 0	1 0 0 0	0 0	1 1 0 0
1 1	1 0 0 0	0 1	1 1 1 0
0 0	1 0 0 1	1 1	0 1 0 0
0 1	1 0 0 1	1 0	0 1 1 0
1 0	1 0 0 1	0 1	1 1 0 0
1 1	1 0 0 1	0 0	1 1 1 0
0 0	1 0 1 0	1 1	0 1 0 1
0 1	1 0 1 0	1 0	0 1 1 1
1 0	1 0 1 0	0 1	1 1 0 1
1 1	1 0 1 0	0 0	1 1 1 1
0 0	1 0 1 1	1 0	0 1 0 1
0 1	1 0 1 1	1 1	0 1 1 1
1 0	1 0 1 1	0 0	1 1 0 1
1 1	1 0 1 1	0 1	1 1 1 1
0 0	1 1 0 0	0 0	0 1 0 0
0 1	1 1 0 0	0 1	0 1 1 0
1 0	1 1 0 0	1 0	1 1 0 0
1 1	1 1 0 0	1 1	1 1 1 0
0 0	1 1 0 1	0 1	0 1 0 0
0 1	1 1 0 1	0 0	0 1 1 0
1 0	1 1 0 1	1 1	1 1 0 0
1 1	1 1 0 1	1 0	1 1 1 0
0 0	1 1 1 0	0 1	0 1 0 1
0 1	1 1 1 0	0 0	0 1 1 1
1 0	1 1 1 0	1 1	1 1 0 1
1 1	1 1 1 0	1 0	1 1 1 1
0 0	1 1 1 1	0 0	0 1 0 1
0 1	1 1 1 1	0 1	0 1 1 1
1 0	1 1 1 1	1 0	1 1 0 1
1 1	1 1 1 1	1 1	1 1 1 1

Input Bits (Stage 2) P_1 P_0	Input State (Stage 2) S_3 S_2 S_1 S_0	Output Bits (Stage 2) O_3 O_2 O_1 O_0	Output State (Stage 2) S_3 S_2 S_1 S_0
0 0	0 0 0 0	0 0 0 0	0 0 0 0
0 1	0 0 0 0	0 0 1 1	0 0 1 0
1 0	0 0 0 0	1 1 0 0	1 0 0 0
1 1	0 0 0 0	1 1 1 1	1 0 1 0
0 0	0 0 0 1	0 0 1 1	0 0 0 0
0 1	0 0 0 1	0 0 0 0	0 0 1 0
1 0	0 0 0 1	1 1 1 1	1 0 0 0
1 1	0 0 0 1	1 1 0 0	1 0 1 0
0 0	0 0 1 0	0 0 1 0	0 0 0 1
0 1	0 0 1 0	0 0 0 1	0 0 1 1
1 0	0 0 1 0	1 1 1 0	1 0 1 1
1 1	0 0 1 0	1 1 0 1	1 0 0 1
0 0	0 0 1 1	0 0 0 1	0 0 0 1
0 1	0 0 1 1	0 0 1 0	0 0 1 1
1 0	0 0 1 1	1 1 0 1	1 0 0 1
1 1	0 0 1 1	1 1 1 0	1 0 1 1
0 0	0 1 0 0	0 1 0 0	0 1 0 0
0 1	0 1 0 0	0 1 1 1	0 1 1 0
1 0	0 1 0 0	0 0 0 0	1 1 0 0
1 1	0 1 0 0	0 0 1 1	1 1 1 0
0 0	0 1 0 1	1 1 1 1	0 1 0 1
0 1	0 1 0 1	1 1 0 0	0 0 1 0
1 0	0 1 0 1	0 0 1 1	1 0 0 0
1 1	0 1 0 1	0 0 0 0	1 0 1 0
0 0	0 1 1 0	1 1 1 0	0 0 0 1
0 1	0 1 1 0	1 1 0 1	0 0 1 1
1 0	0 1 1 0	0 0 1 0	1 0 0 1
1 1	0 1 1 0	0 0 0 1	1 0 1 1
0 0	0 1 1 1	0 0 0 1	0 0 0 1
0 1	0 1 1 1	0 0 1 0	0 0 1 1
1 0	0 1 1 1	1 1 0 1	1 0 0 1
1 1	0 1 1 1	1 1 1 0	1 0 1 1

Input Bits (stage 2) P_1 P_0	Input State (Stage 2) S_3 S_2 S_1 S_0	Output Bits (Stage 2) O_3 O_2 O_1 O_0	Output state (Stage 2) S_3 S_2 S_1 S_0
0 0	1 0 0 0	1 0 0 0	0 1 0 0
0 1	1 0 0 0	1 0 1 1	0 1 1 0
1 0	1 0 0 0	0 1 0 0	1 1 0 0
1 1	1 0 0 0	0 1 1 1	1 1 1 0
0 0	1 0 0 1	1 0 1 1	0 1 0 0
0 1	1 0 0 1	1 0 0 0	0 1 1 0
1 0	1 0 0 1	0 1 1 1	1 1 0 0
1 1	1 0 0 1	0 1 0 0	1 1 1 0
0 0	1 0 1 0	1 0 1 0	0 1 0 1
0 1	1 0 1 0	1 0 0 1	0 1 1 1
1 0	1 0 1 0	0 1 1 0	1 1 0 1
1 1	1 0 1 0	0 1 0 1	1 1 1 1
0 0	1 0 1 1	1 0 0 1	0 1 0 1
0 1	1 0 1 1	1 0 1 0	0 1 1 1
1 0	1 0 1 1	0 1 0 1	1 1 0 1
1 1	1 0 1 1	0 1 1 0	1 1 1 1
0 0	1 1 0 0	0 1 0 0	0 1 0 0
0 1	1 1 0 0	0 1 1 1	0 1 1 0
1 0	1 1 0 0	1 0 0 0	1 1 0 0
1 1	1 1 0 0	1 0 1 1	1 1 1 0
0 0	1 1 0 1	0 1 1 1	0 1 0 0
0 1	1 1 0 1	0 1 0 0	0 1 1 0
1 0	1 1 0 1	1 0 1 1	1 1 0 0
1 1	1 1 0 1	1 0 0 0	1 1 1 0
0 0	1 1 1 0	0 1 1 0	0 1 0 1
0 1	1 1 1 0	0 1 0 1	0 1 1 1
1 0	1 1 1 0	1 0 1 0	1 1 0 1
1 1	1 1 1 0	1 0 0 1	1 1 1 1
0 0	1 1 1 1	0 1 0 1	0 1 0 1
0 1	1 1 1 1	0 1 1 0	0 1 1 1
1 0	1 1 1 1	1 0 0 1	1 1 0 1
1 1	1 1 1 1	1 0 1 0	1 1 1 1