

Memetic Algorithm with Population Management for the Two-dimensional Loading Vehicle Routing Problem with Partial Conflicts

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Abstract: The two-dimensional loading vehicle routing problem with partial conflicts combines two NP-hard problems: the capacitated vehicle routing problem (CVRP) and the two-dimensional bin-packing problem with partial conflicts (2BPPC). This problem arises for example in hazardous waste collection, where some materials can be partially conflicting. In this paper, we propose a memetic algorithm with population management to resolve this new problem. A modified SHF-D heuristic is used to obtain feasible packing in each vehicle. The proposed approach is tested on a new benchmark, created by adding partial conflicts to instances from the literature.

1 INTRODUCTION

Some recent works have been dedicated for coupling vehicle routing and items packing in the vehicles to obtain solutions for the two-dimensional loading capacitated vehicle routing problem (2L-CVRP). This problem was proposed for the first time by Iori et al. (Iori, 2004) who developed an exact method (Iori et al., 2007) to solve it. Several approximate methods were proposed by Gendreau et al. (Gendreau et al., 2008), Zachariadis et al. (Zachariadis et al., 2009), Leung et al. (Leung et al., 2011), Fueller et al. (Fueller et al., 2009) and Duhamel et al. (Duhamel et al., 2011). In all previous studies, the authors solved a basic two-dimensional bin-packing problem for each vehicle.

Khebbache et al. (Khebbache et al., 2011) studied the 2L-CVRP with time windows. The authors proposed six heuristics and metaheuristics for this variant. The three dimensional case referred to as 3L-CVRP with time windows was also investigated by Moura et al. (Moura and Oliveira, 2009) and (Moura, 2008).

The concept of distance constraint was introduced by several authors in studies dedicated to packing problems. In (Stoyan and Yaskov, 1998), each item has to be separated from all the other items and from the bin borders by a given distance, depending on the item. Therefore, there is no contact between the edges of all items. These authors (Stoyan and Yaskov,

1998) proposed a mathematical model and a combination of a branch and bound and a reduced gradient method to solve this problem. They considered both the cases where the items are either rectangles or circles. The problem was generalized to the three dimensional case by Stoyan and Chugay (Stoyan and Chugay, 2009). In this last study, items are either cylinders or parallelepipeds and the packing area is considered of some given shape. In (Beaumont et al., 2010), the maximal distance between two items packed in the same bin has to be smaller than a given threshold.

In some industrial applications such as in Hazardous Material (HazMat) transportation, residues of HazMat are first decanted into rectangular or circular containers at their source, then the schedule of the containers transshipment to reprocessing plants is organized. According to HazMat classification, some products can be *partially conflicting*. In this case these materials can be stored or shipped together, but they have to be separated by a given safety distance. Some HazMat can also be in *total conflict*, it is then not permitted to store them in the same warehouse or transport them in the same vehicle. In HazMat transportation legislation, one horizontal meter distance have to be kept between any two partially conflicting materials. This industrial application motivated us to introduce the two-dimensional bin-packing problem with partial conflicts (2BPPC), for which a mathematical model, several heuristics and a multi-start genetic

algorithm were proposed (Dhaoui et al., 2012).

This paper provides a first study for the two dimensional loading with partial conflicts capacitated vehicle routing problem (2LPC-CVRP). The resolution approach proposed here combines a memetic algorithm for the routing problem and a heuristic for the packing. In Section 2, the two problems (vehicle routing and 2-dimensional bin-packing with partial conflicts) are presented. In Section 3, the different components of the memetic algorithm are explained. Section 4 is dedicated to computational results and a conclusion ends this paper.

2 PROBLEM PRESENTATION

2.1 Vehicle Routing Problem

The capacitated vehicle routing problem can be defined on an undirected graph $G = (V, E)$ with $V = \{0, 1, \dots, n\}$ the set of vertices and E the set of edges. The vertex 0 represents the depot, where a fleet of K identical vehicles with capacity Q is provided, whereas the other vertices represent the customers. A demand d_i is associated to each customer i and a traveling cost c_{ij} is associated to each edge $[i, j]$.

Given the graph G , the vehicle routing problem consists in finding the routes satisfying the following constraints: each route starts and ends at the depot, each customer is visited only once by a single vehicle and the total demand of customers assigned to the same route can not exceed the vehicle capacity Q . The aim is to minimize the total traveling cost. The objective function is defined as follows:

$$\sum_{k=1}^K \sum_{i=1}^n \sum_{j=1}^n c_{ij} \cdot x_{ij}^k \quad (1)$$

Where x_{ij}^k is a binary variable that takes the value 1 when the edge $[i, j]$ is crossed by the vehicle k going from i to j .

2.2 Two-dimensional Packing Problem with Partial Conflicts

This problem has been studied for the first time by Dhaoui et al. (Dhaoui et al., 2012). A 2BPPC instance consists in a set $A = \{1, \dots, n\}$ of items which have to be packed in a minimum number of identical bins. A bin is defined by its height H and its width W . An item i has a height h_i and a width w_i ($h_i, w_i \in \mathbb{N}$) and may be in partial conflict with some other specified items. A solution of the problem consists in assigning each item i to a bin and defining its position, denoted by (x_i, y_i) which corresponds to the coordinates

of its bottom left-hand corner in the bin, without overlapping while keeping a safety distance D between partially conflicting items (Figure 1). The considered distance is denoted d_∞ and is defined as follows: let i and j be two items inserted in the same bin and assume that their positions in the bin are: (x_i, y_i) and (x_j, y_j) . $d_\infty(i, j) = \max\{|x_i - x_j|, |y_i - y_j|\}$.

In this study, the distance d_∞ is chosen because it is the one often used in hazardous material transportation. In this sector, the guideline is to keep a prefixed horizontal or vertical spacing between any two materials partially conflicting. For example, corrosive material and flammable solid material are in partial conflict as indicated in Hazardous material classification data-base (Environment Canada, 2002). hence, when they have to be packed within the same bin (a storage area or a vehicle), a safety distance of 1 horizontal or vertical meter must separate them. Figure 1 shows the difference on using the distance d_∞ and the Euclidean distance d .

This study is dedicated to a new problem, the 2LPC-CVRP which is a combination of both vehicle routing optimization and two-dimensional packing of items with partial conflicts. In this work, non-sequential (unrestricted) packing without items rotation is considered.

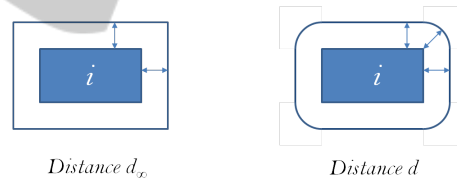


Figure 1: Safety distance.

In Figure 2, items 1 and 2 are conflicting with item i , that is why a safety distance D is kept between them.

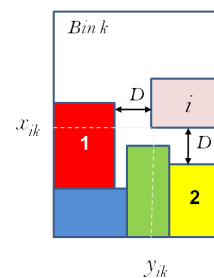


Figure 2: Partial conflicts.

3 MEMETIC ALGORITHM WITH POPULATION MANAGEMENT

In this section, the different components of our memetic algorithm (MA) are detailed. First, an initial population of NP chromosomes is generated. A chromosome is coded as a sequence of n customers (a permutation) without route delimiters, which can be seen as a giant tour violating the capacity and packing constraints. A feasible solution of our problem is obtained by cutting this giant tour into feasible routes using a *Cutting Procedure*. The parent solutions are selected with a binary tournament and crossed over to obtain child solutions or *offsprings*. A local search is applied with a given probability to improve the solutions quality. Finally, the acceptance or rejection of new solutions is decided by the population management procedure. The best NP solutions constitute the new population. This treatment is repeated NG times.

3.1 Initial Solutions

A first solution is computed with an adaptation of Clarke and Wright heuristic (Clarke and Wright, 1964) and is then converted to a chromosome by concatenating its routes. The resulting chromosome is included in the initial population which is completed by generating randomly the $(NP - 1)$ remaining elements. Each chromosome in the population is evaluated using the cutting procedure explained in section 3.3.

3.2 Crossover

Parents are selected by binary tournament method: two chromosomes are randomly selected in the population and the best one is the first parent. This process is repeated to get the second parent. Then they are combined using the classical operator OX for *Order Crossover* (Goldberg, 1989). OX begins by randomly selecting two break points i and j ($1 < i \leq j < n$) in the first parent. Customers located between the positions i and j in the first parent are copied into the child chromosome in the same positions. The remaining customers are added by browsing the second parent from left to right and are copied in the order of their appearance. Each crossover provides a single child. The fitness of the obtained child solution is then computed using the *Cutting Procedure*.

3.3 Cutting Procedure

This procedure starts by applying the method called *Split* proposed by Prins (Prins, 2004) for the CVRP.

Split consists in computing a shortest path in an auxiliary graph containing a dummy vertex 0 and a vertex for each customer. The packing feasibility of the obtained routes is checked with a modified version of SHF-D heuristic. Infeasible routes (for the packing problem) are concatenated to obtain a new giant route (or a sub-chromosome). For this new sequence, a new packing heuristic is performed: customers are assigned to routes in the order given by the sub-chromosome. When the customer's demand can't be packed in all the available vehicles, a new route is created and this later is initialized with this customer. A feasible solution for the 2LPC-CVRP is obtained at the end of this procedure.

3.4 Population Management

To avoid premature convergence of the population to a local minimum, it is possible to use a management tool that allows to choose whether to keep or reject a solution according to a distance criterion from the existing solutions. Memetic algorithms with population management have been introduced by Sörensen and Sevaux (Sörensen and Sevaux, 2006). We consider the distance computing the number of "broken pairs" as introduced in (Martí et al., 2005). Let consider two chromosomes X and Y . The customer that occupies the position i in the chromosome X is denoted X_i . The distance $D(X, Y)$ equals the number of pairs $\{X_i, X_{i+1}\}$ that are not adjacent in Y . The distance of a chromosome to a population P is defined as follows:

$$D_P(Y) = \min\{D(X, Y) : X \in P\} \quad (2)$$

A new solution Y is accepted in the population P if its distance from the population is greater than a given level Δ . When Δ equals 0, it simply consists in eliminating copies. The management procedure starts by sorting the chromosomes of a population P in a decreasing order of their quality, which corresponds to increasing order of costs. The population size is constant, that means when a new solution is accepted, another solution is eliminated. To maintain the best found solutions, the solution to be eliminated is selected randomly in the worst half of the population. The level Δ is then updated based on the rate of refusal or acceptance of solutions (when more than M solutions are accepted, Δ is increased by one unit, and when more than M solutions are rejected, Δ is decreased by one unit).

3.5 Modified Shelf Heuristic Fill-Dynamic (SHF-D)

This heuristic was introduced by Ben Messaoud et al. (BenMessaoud et al., 2004) and is mainly used for guillotine cutting problems. In this kind of problems, the cutting tool has to go from one edge of the rectangle (or the strip to cut) to the other. Shelf algorithms allow to resolve this problem. Packing is obtained by inserting items from the left to the right side, while forming shelves. The first shelf corresponds to the rectangle bottom. The next shelf bottom is obtained with the horizontal line that coincide with the top of the highest item in the previous shelf. The treatment is repeated for all items. The modified version presented in this section was proposed for the 2BPPC by Dhaoui et al. (Dhaoui et al., 2012).

In the classical version, items are sorted out in a decreasing order of heights. In this modified version, items are separated into sets corresponding to their classes (as in hazardous materials classification, items are assigned to classes and the partial conflicts are rather defined between classes than items). Then they are inserted in a way that the items of two classes treated successively are not conflicting. If it is impossible to obtain an order that separates every pair of conflicting classes, a safety distance is kept when necessary.

The procedure that sorts out the classes starts by assigning the first position to the class that has the biggest degree of conflicts. The degree of conflicts of a class corresponds to the number of classes conflicting with it. After adding a class to the list, the following class is chosen among the classes non conflicting with it.

The procedure sorting out the classes is described in Algorithm 1. *Classes* stands for the table of classes and represents the input of the algorithm, while *List* is the output corresponding to the new order of the classes. *Nc* is the number of classes and *c* and *ind* are indexes of the classes. *Degree* is a table in which are recorded the degree of the classes. Finally, *Compatible* is a boolean that takes the value *true* if the considered classes are compatible, and the value *false* if not.

Each item is inserted in the lowest possible position left-justified in the first shelf where it fits entirely, if any. If none of the existing shelves can contain it, a new shelf is initialized. This heuristic is different from other classical shelf algorithms with its tendency to reuse at best the free areas in each shelf, by inserting items on top of the others while preserving guillotine cutting constraints.

At any stage of the heuristic, each used bin con-

Algorithm 1: Sorting out classes Procedure.

```

Sort out Classes in a decreasing order of Degree
c = 1
List(c) = Classes(c)
Repeat
  ind = c + 1
  Compatible = false
  Repeat
    If (Classes(ind) conflicts with List(c)) then
      ind = ind + 1
    Else Compatible = true
  Until ((Compatible = true) or (ind = Nc))
  If (Compatible = true) then List(c) =
    Classes(ind)
  Else List(c) = Classes(c + 1)
  c = c + 1
Until (c = Nc)

```

tains a set of empty spaces. These spaces are available rectangles; each of them is defined with its height, width and coordinates of its bottom left corner. To place an item, the method proceeds by examining each available rectangle in turn and finding whether the item fits into it and whether it is conflicting with the already packed items that surround the considered rectangle. Rectangles are examined in an increasing order of the ordinates of their bottom left corner. In case of equality, abscissa is considered and the rectangle with lowest value of abscissa is examined first.

In Figure 3, after the insertion of item 1, the used available rectangle is deleted while two new available rectangles are created: *Top* ($W_1 = W, H_1 = H - y_1$) is the available rectangle on top of the inserted item and *Right* ($W_2 = W - x_2, H_2 = H$) is the available rectangle on its right.

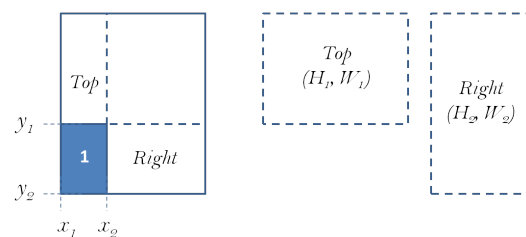


Figure 3: Available rectangles.

The procedure *Update* is described in Algorithm 2. *Nr* is the number of available rectangles. Let *p* be the index of the available rectangle where the current item is inserted. If there is an intersection between an existing rectangle *r* and the rectangle *p*, the dimensions of the rectangle *r* are modified in order to eliminate the intersection area. If the new height or

width of the treated rectangle becomes null, the rectangle is deleted. Two new rectangles are created (*Top* and *right*) and the rectangle *p* is deleted.

Algorithm 2: Procedure to update available rectangles.

```

Create Rectangles (Top) and (Right)
For  $r = 1$  to  $Nr$ 
    If  $Rectangle(r) \cap Rectangle(p) \neq \emptyset$ 
        Update  $Rectangle(r)$ 
    If ( $W_r = 0$  or  $H_r = 0$ ) Delete  $Rectangle(r)$ 
end for
Delete  $Rectangle(p)$ 
Sort out  $Rectangle$ 
    
```

In the dynamic version of the method (SHF-D), a shelf height is not fixed with the height of the first inserted item. A temporary height is updated after each insertion. Let Nr be the number of available rectangles in the current bin and Ns the number of shelves within it. If the current item does not fit in all the $Nr - 1$ available rectangles and the last rectangle has to be used (the rectangle with the biggest ordinate of bottom-left corner), a new shelf is initialized with this item and the height of the previous shelf is fixed. In Algorithm 3, let p be the index of the valid rectangle and Y_p its ordinate.

Algorithm 3: Fix shelf height Procedure.

```

If ( $p = Nr$ ) then
    Shelf Height( $Ns$ ) =  $Y_p$ 
     $Ns = Ns + 1$ 
    Initialize Shelf( $Ns$ ) with Current Item
else
    Insert Current Item in Shelf( $Ns$ )
End if
    
```

3.6 Local Search

Three kinds of moves are used, each of them defining a neighborhood structure. For the moves that concern two different routes, the packing feasibility is checked with modified SHF-D and only feasible moves are performed. The moves are sorted in increasing order of complexity and are:

Move 1: Customer Relocation

This move type relocates a customer from his current position to another position. The move can be used for every possible insertion position, and can involve only one route or two different routes.

Move 2: Position Exchange

This move type exchanges the routes that cover a

pair of customers. The two customers swap their positions in the routes too. It is employed for every pair of routes and for every customer pair involved in the considered routes. When the move is performed within a single route, the customers swap their positions within this route.

Move 3: 2-opt

The move 2-opt is defined by two customers a and b . Let X be a route represented as a list of customers containing a copy of the depot in the beginning and at the end, $X = (x_1 = 0, x_2, \dots, x_i, x_{i+1} = 0)$. Only the case where a and b are within the same route X with $x_i = a$, $x_j = b$ and $i \leq j$ is considered. The move 2-opt consists in inverting the subsequence $(x_i, x_{i+1}, \dots, x_j)$. It is replaced by the subsequence $(x_j, x_{j-1}, \dots, x_i)$.

4 EXPERIMENTAL RESULTS

The used computer is a PC Pentium(R) Dual-Core CPU T4400 2.20 GHz with an operating system Windows 7. For our tests, we adapted the instances of Iori (Iori, 2004) created for the 2L-CVRP. In these instances, the number of items and their sizes are created in five sets. The first set corresponds to instances of a capacitated vehicle routing problem (CVRP). In the set i , with $i = 1, \dots, 5$, the number of items assigned to each customer ranges from 1 to i . Dimensions of items are generated according to uniform distribution in given intervals (Iori et al., 2007). In these instances the number of customers ranges from 15 to 35, while the number of items is between 15 and 114.

In order to obtain 2LPC-CVRP instances, each item is assigned to one of 5 categories of hazardous materials and a class index is then associated randomly to each item. Partial conflicts are generated randomly between some classes. The obtained instances are constituted of 16 groups of instances, each one contains five files.

For the main parameters setting, a design of experiments "Latin Square" with 9 situations and 3 levels was used. The parameters are fixed as follows: $(NG, NP, \alpha) = (20, 20, 1)$, where NG is the number of generations, NP is the population size and α is the rate of local search.

The results on the instances considering the conflicts into account are given in the table 1. The aim is to minimize the overall traveling cost (as in the classical CVRP) when assuming an unlimited number of vehicles. The obtained total cost and the computing time are reported.

The algorithm was also tested on the instances without conflicts in order to estimate its performance when compared to the best results of the literature ob-

Table 1: Average results of our MA per group of instances.

Group	Cost	Time	Group	Cost	Time
1	327,85	1,61	9	677,97	4,32
2	370,89	2,80	10	725,66	5,12
3	427,45	1,84	11	683,89	3,84
4	403,49	4,12	12	2232,56	5,66
5	483,41	3,94	13	1593,85	5,78
6	643,60	3,50	14	1308,52	7,14
7	687,91	3,92	15	851,64	4,96
8	848,06	4,10	16	708,10	3,50

Table 2: Average results for set 1 (CVRP) of Iori instances (groups from 1 to 16).

Groups	Best		MA	
	Cost	Time	Cost	Time
1	278,73	0,11	278,73	3,1
2	334,96	0,02	334,96	0,04
3	358,85	0,69	358,85	2,8
4	430,88	0,83	430,88	2,1
5	375,28	0,13	375,28	3,4
6	485,85	0,66	495,85	1,3
7	568,56	0,25	568,56	4
8	568,56	0,19	568,56	3,5
9	607,65	3,97	607,65	4,7
10	535,8	0,59	558,27	4,1
11	505,01	0,8	531,30	4,2
12	610,00	12,6	646,46	3,6
13	2006,3	0,83	2248,21	4,4
14	837,67	0,74	913,18	4,1
15	837,67	1,36	1136,70	2,3
16	698,61	22,8	708,42	4,3

Table 3: Average results for sets from 2 to 5 (2L-CVRP) of Iori instances (groups from 1 to 16).

Groups	Best		MA	
	Cost	Time	Cost	Time
1	284,42	0,90	286,89	1,3
2	339,26	0,10	339,26	1,40
3	376,32	0,50	381,34	2,5
4	435,01	0,20	443,25	1,1
5	379,04	0,10	401,01	5,00
6	497,04	0,40	506,13	4,00
7	691,11	1,40	720,79	3,8
8	678,84	0,80	747,32	3,9
9	612,01	0,60	625,50	1,9
10	675,79	15,10	744,65	5,6
11	705,95	11,30	771,83	5,6
12	611,26	16,90	663,18	4,3
13	2490,62	78,00	2859,10	5,9
14	984,42	79,90	1199,27	7,1
15	1144,69	257,70	1275,25	7,7
16	699,79	6,00	709,75	5,1

tained by Duhamel et al. (Duhamel et al., 2011). For this purpose, a restriction on the number of vehicles is added. We consider non-sequential packing without items rotation. In average, our MA has a distance

of 6% from the best known solutions.

In Table 2, the results on the first group of instances are reported. this group corresponds to a pure transportation problem. Our algorithm finds the best known solutions for 8 out of 16 instances. The computation time of our MA does not exceed 5 seconds, while it may be much higher for some results obtained by Duhamel et al. (Duhamel et al., 2011).

Table 3 gives the average results for sets from 2 to 5. Our algorithm finds the best known average only for set 2. For the other sets, our results are very close to the best known results in the literature, particularly for instances of the first groups (groups 1 to 6). For 8 groups out of 16, the gap is less than 4%. For 7 other groups the gap ranges from 6% to 14%. Only the gap of the group 14 is over 20%. The average gap obtained for all studied instances to the best known solutions is about 6.6%.

5 CONCLUSIONS

This paper presents a first study for the two-dimensional loading with partial conflicts capacitated vehicle routing problem. A memetic algorithm with population management is designed to solve routing part, while a heuristics is used to solve packing problem. The first results are promising. Further results would be given in the conference.

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