A Topological-Geometrical Pipeline for 3D Cracking-like Phenomena

Jérémy Riffet[1], Nicolas Castagne[1], Emmanuelle Darles[2] and Annie Luciani[3]
[1]*Univ. Grenoble Alpes, Grenoble INP, Institute of Engineering, ICA, 38000 Grenoble, France*
[2]*Univ. Poitiers, CNRS, XLIM, UMR 7252, F-86000 Poitiers, France*
[3]*ACROE, Grenoble, France*

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Abstract: Animation of one-to-many phenomena (fractures, tears, breaks, cracks…) is challenging. This article builds over recent works that proposed a 3-stages modelling and simulation pipeline, made of a cascade of models: geometry-free physical model → explicit modelling of the evolving topology → geometrical model. On the Physics' side, in the framework of masses-interactions network modelling, the article extends the recent Splitting-MAT method, where the physical splits occur onto the material points, toward 3 dimensional volume models. Downstream, it introduces a topo-geometrical pipeline adapted to this upstream split-on-the-masses property. Experiments, and analysis of the complexity of the topo-geometrical part, show that, while offering constructible and manageable means, separating Physical, Topological and Geometrical aspects in the 3-stages pipeline enables a rich variety of one-to-many dynamics, with good efficiency.

1 INTRODUCTION

Fracture, tearing, breaking, cracking, or more generally one-to-many visual dynamics featuring topological transformations are attractive, but their modelling and simulation are challenging. Research has recently worked on two categories of approaches.

The first approach focuses on extending "geometry-based" physical methods, such as Finite Elements, Mass-Spring Meshes or Diffuse Elements methods, toward fracturing, tearing, breaking, etc. In such cases, the physical algorithms are embedded into a geometrical mesh. When a topological transformation occurs, both the geometrical and physical models need to be co-transformed with one another. Managing globally such geometrico-physical remeshing process is difficult. (Muguercia et al., 2014) and (Frerichs et al., 2015) offer surveys of recently proposed solutions and models. In parallel with the present work, one can also note that it has recently been proposed to root on an explicit representation of the topological aspects, with the aim to manage with more control and systematism the physico-geometrical remeshing issues (Carter et al., 2000; Meseure et al., 2010; Fléchon et al., 2013; Paulus et al., 2015).

In the second category of approaches, to which this article relates, modelling and computation of the splittable dynamics root on "geometry-free" (or "mesh-free", or "morphology-free") physics-based methods, such as: masses-interactions networks modelling (Jund et al., 2012; Luciani and Godard, 1997), meshless techniques (Zhuang et al., 2012; Steinemann et al., 2009; Pauly et al., 2005), smoothed particles hydro-dynamics (Chen et al., 2013), frame-based simulation (Manteaux et al., 2015), and others.

In these cases, the physical model is not tied to a pre-existing geometrical mesh, and does not express matter contiguity. This eases the modelling and computing of the physical state changes: the physical model can be thought of, computed, without the burden of geometrical aspects.

Anyhow, when working with Geometry-free physics-based approaches, the dynamics is most often generated as a set of moving points. Consequently, a geometrical model must be set up to visualize such punctual movements. In the case of one-to-many phenomena, the physical state changes must also control downstream topological transformations within the geometrical model.

To address this issue, (Zhuang et al., 2012) build over the principles of level sets. (Pauly et al., 2005) propose employing small elliptical surfaces called

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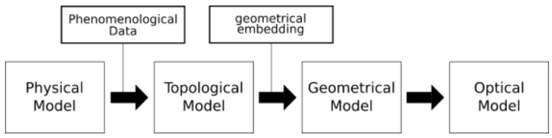


Figure 1: A cascading pipeline for the modelling and simulation of one-to-many phenomena.

surfels. (Steinemann et al., 2009) and (Chen et al., 2013) employ an adjacency graph set up over the moving particles.

Another recent proposal consists in introducing a model explicitly dedicated to the topological aspects. Besides first experiments in (Darles et al., 2011), (Jund et al., 2012) and (luciani et al, 2014) position a global modelling and simulation pipeline, made of a cascade of submodels, from upstream to downstream (Figure 1): a physics-based model → a topological model → a geometrical model. Employing a central topological model helps managing efficiently, systematically and formally the topological transformation, under control of the physics-based punctual movements. Also, segmenting the complete model in layers of submodels makes it possible to tie each of the models to a clearly delimited part of the final visual phenomena: splittable dynamics thanks to the physical model; management and transformations of the relations thanks to the topological model; geometry thanks to the geometrical model.

Recently, in the context of Geometry-free physics-based approaches, (Kalantari et al., 2014) extended masses-interactions networks modelling, by introducing the Splitting MAT system. This system allows the splits to occur directly on the material points, though without any computational overhead. However, Splitting MAT have not yet been employed within the global Physics → Topology → Geometry pipeline.

This article extends the original pipeline proposed in (Jund et al., 2012) and related articles to the cases where Splitting-MAT (Kalantari et al., 2014) are employed on the upstream Physics' side. Our contributions are:

1/ On the physics-based side, we extend the Splitting-MAT method to the case of splittable three-dimensional volume models.

2/ Downstream, we introduce a topological-geometrical coating pipeline adapted to the splitting-MAT principles in 3D. We explain how the upstream split-on-the-masses property eases setting up the topo-geometrical process downstream.

3/ We present various experiments on fractures, breaks and tears phenomena. We show that the pipeline allows modelling and simulating a range of fracturing/cracking/tearing effects with good efficiency, possibly at interactive frame-rate.

2 THE TOPO-GEOMETRICAL PIPELINE

This section presents the proposed physical→topological→geometrical pipeline built over the Splitting MAT method. Section 2.1 summarizes the principles of the Splitting MAT physics-based system, and discusses the upstream physical model with Splitting MAT. Section 2.2 covers the data exported from the physics toward the topological, then geometrical stages. Section 2.3 provides details on the G-Map topological system we employ. Section 2.3 covers the topo-geometrical pipeline downstream: as originally proposed in (Jund et al. 2012), we present successively the *Construction*, *Association*, *Modification* and *Affectation* steps.

For more clarity, throughout this section, explanations are first based on an exemplary 2D model, which is globally summarized on Figure 3. In each paragraph, we briefly explain how the pipeline can be evolved to 3D volume models. The experimental models presented in section 3 are all 3D volume models.

2.1 Physical Model with Splitting MAT

The Splitting-MAT methodology extends the possibilities of masses-interactions networks modelling in regards to one-to-many phenomena (fractures, breaks, tears…), by enabling the splits to occur directly *onto* the MATerial points – the masses of the network. The method has a fully constant algorithmic complexity, and guarantees by construction the stability of the physics, no matter how the model evolves (break, tear, split…) during simulation. (Kalantari et al., 2014) provides details on the system and its stable computing cost.

Modelling with splitting–MAT starts by defining the smallest possible physical entities, corresponding to the fully-split state, by interconnecting some masses with physical interactions. Then, masses of various entities are united into *Mass-unions*. A Mass-union is created by tying its masses one to another with *Duplets*. As long as a duplet remains active, the 2 tied masses remain in the same Mass-union, and will keep the same exact behaviour: same position, same speed. Each Duplet is associated with a *Sensor* which, when triggered, inactivates the Duplet.

Depending on the emerging phenomena, duplets may be progressively inactivated, leading to Mass-unions' splits. Hence, masses may progressively gain their autonomy, possibly down to the fully-split state.

When working with 2D models, each elementary physical entity could be made of 3 or 4 masses. Figure 3 (a, b, c, left side) explain how the physical model could be built while modelling, and split during simulation, in the case of a very simple exemplary 2D model made of only 4 elementary physical entities, 13 masses, and 5 Mass-unions.

In this work, on the physics-based side, we extend the Splitting-MAT method to 3D volume models – where only surfacic models where presented in (Kalantari et al, 2014). To work in 3D, each elementary physical entity can be made of 8 masses, and various interactions, that form a hexahedral physical entity (Figure 4a). Then, up to 8 masses taken from 8 different smallest physical entities are tied into initial Mass Unions, by using a minimum of 7 duplets (Figure 4b). During simulation, the splitting process is then exactly the same as in 2D, even though each mass has a 3D position and velocity.

2.2 Data Passed to the Topo-geometric Stage

In our proposed pipeline, only phenomenological data generated from the physics-based model are passed to the topo-geometrical level. Two categories are considered in this work.

The first consists in the punctual movements of each of the masses of the physical model, sampled at the physics' simulation frequency, no matter they are gathered in Mass-unions or not. Each of these punctual movements is called an *evolution function* (Luciani et al., 2014).

The second consists in the state of the Mass-unions. This data stream is new as compared to (Luciani et al., 2014), which did not employ Splitting MAT. The Mass-unions' state data stream is event-based: whenever a Mass-union splits in the physics, the indexes of the masses forming the newly created Mass-unions are passed to the downstream model along with the date of the event.

2.3 Adjacency Graphs Fundamentals

In order to handle easily and formally topological constructs and modifications in large volume sets, we employ a structure that stores cells adjacencies. There exist many graph models in the literature, such as for example half-edges graphs.

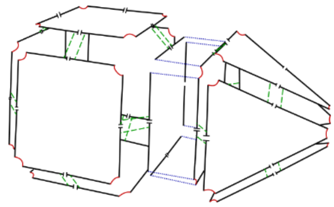


Figure 2: Two darts sewed by α_0 link form an edge (black lines). Several edges sewed by α_1 links (red curves lines) form a face. Several faces sewed by α_2 links (green large dotted lines) form a volume. Several volumes sewed by α_3 links (blue thin dotted lines) form one connected component.

In this work, we employ the generalized map formalism (Lienhart, 1994). Each cell in dimension N (N>0) is created by sewing different N-1-D cells to obtain a N-D cell. Hence, a 1D cell (topological vertex) is created by sewing 0D elementary cells called "darts". Sew operations are mathematically defined as bijective functions α_i (with *i* the dimension of the sew operation) called involutions. The N-D topological cells are the nodes of the adjacency graph, and the involutions represent its edges. Topological cells, such as topological vertices, edges or faces, then correspond to a set of darts that are sewed with each other's with chosen involutions, called *orbit* (Figure 2).

Interestingly, the system enables finding any adjacency relations quickly and automatically, by using simple graph scanning. Additionally, it builds on generic principles to guarantee consistency and coherency of the topology during construction, and whenever performing any topological modification.

2.4 Topo-geometrical Pipeline

Downstream Physics, to finally obtain a visible evolving geometry, the proposed topo-geometrical pipeline roots on 4 steps: in first, during the modelling stage, construction and association steps. Secondly, during the simulation stage, modification and affectation steps.

2.4.1 Construction

The first step, called *Construction*, consists in building an initial topology, which will be the core of the entire topo-geometrical pipeline and will be evolved during simulation.

As compared to (Kalantari et al, 2014), employing Splitting MAT in the Physics makes it possible to build a simple base topology. This is a important advantage as compared to previous works on the 3 stages pipeline that did not employ Splitting-MATs.

This base topology can be obtained simply by employing a building process similar to the physical model's building process.

Figure 3a shows how the base initial topology obtained in the case of simple use-case 2D model. It is made 4 topological polygonal faces, sewed with each other to form a single large polygonal surface. Hence, each 2D elementary physical entity corresponds to a polygonal face.

When working with 3D volume models, the building of the base initial topology follows the same process. Though, instead of leading to polygonal faces, it results in tetrahedral topological volumes, sewed in α_3. Each of these volumes corresponds to an elementary entity in the physical model. Noticeably, besides the *base* topology discussed in this article, it would be equally possible to experiment with other topologies (e.g. refined).

2.4.2 Association

Association consists in bijectively associating each evolution function to one or several elements in the topological structure (darts, orbits, etc.).

Employing Splitting-MATs on the physical level enables a cunning association strategy, as compared to previous works without Splitting MATs. This association is achieved by traversing the topological structure in the same order the physical model was built. During this scan, the evolutions functions are associated one after another to a selected orbit in the topological model.

In 2D, the proposed association roots on the notion of *face's corner*, topologically defined as the set of darts in the orbit$< \alpha_1 >$ of a chosen dart. Figure 3b illustrate the resulting Association in the case of the simple 2D exemplary model. Each evolution function (each moving material point) is associated with a single face's corner orbit in the topological structure. For example, the topological vertex in the centre of the topological model is made of 4 face's corners, each one gathering 2 darts. These 4 face's corners are associated to the 4 corresponding evolution functions output from the physics model. It should be reminded that, since masses will keep the same exact position as long as they remain in the same Mass-Union, the corresponding face's corners will be associated downstream to this unique position, until a split occurs in the Physics.

The proposed association strategy extends rather simply to 3D, by considering the notion of *volume's corner*, instead of face's corner. A volume's corner is topologically defined as the set of darts in the orbit$< \alpha_1 , \alpha_2 >$ of a chosen dart. Hence, as we use tetrahedral volumes in the Experiment section, we simply have to associate each evolution function with a single volume's corner (instead of a face's corner).

In G-Map data structure, storing associations in the topology is achieved by storing the evolution function's index in the orbit. Constant time access is achieved from any dart of the orbit to the value (i.e position) of the associated evolution function.

2.4.3 Modification

During simulation, the *Modification* step consists in progressively transforming the topological model, to implement topologically fractures and splits under control of the Physics. As compared to (Luciani et al., 2014), employing splitting MATs upstream enables a more direct control of the topological transformations, thanks to the events received whenever a Mass-union splits in the Physics. In the case of our 2D simple exemplary model, the modification process would simply consist in unsewing from each other's the face's corners which *ids* are not any more in the same Mass-union (figure 3c). In this process, the GMap's implementation automatically maintains the consistency of the topology, so avoid non-manifold topology. As a consequence, in 2D, unsewing two face's corners ultimately, and automatically, results in separating topological edges.

When working with 3D volume models, a Mass-union splits results in separating the two corresponding volume's corners (instead of face's corners). Even in the third dimension, the GMap's implementation ensures the consistency of the topology. Consequently, unsewing the 2 volume's corners automatically unsews all the α_3 links between the adjacent darts of these corners. This results in separating faces of two adjacent topological volumes. We can retrieve in constant time the involutions to unsew in response to a split event received from the physical model by the simple correspondences between the upstream physical masses and the topological face's corners in 2D (or volume's corners in 3D) of the base topology.

2.4.4 Affectation and Geometrical Model

During simulation, *Affectation*, consists in setting up on each rendering step a visible geometry, by embedding geometrically the topological map. With this step, the one-to-many phenomena are finally shaped to the eye.

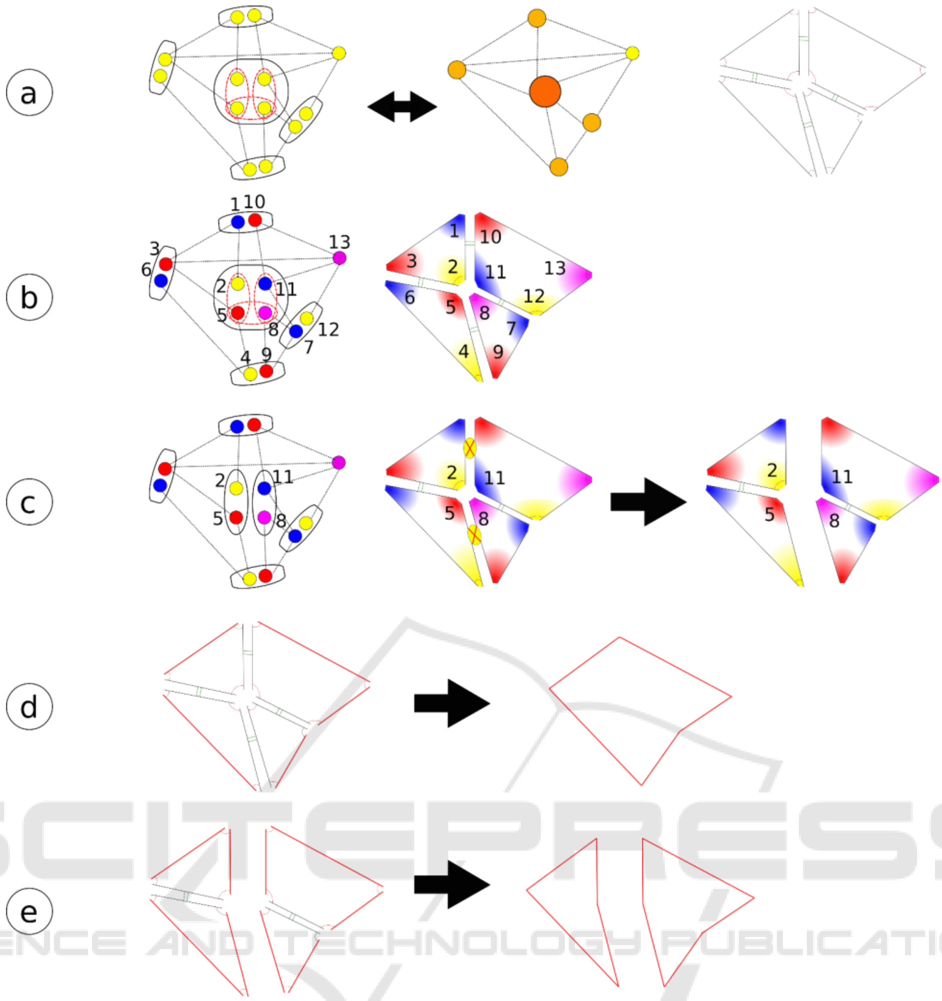
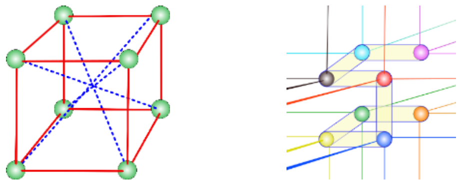


Figure 3. Simple exemplary 2D model illustrating the topo-geometrical Pipeline's Steps. **(a) Construction process**. Left figure: elementary physical entities, made of masses (yellow circles) linked by interactions (dot lines), and Mass-Unions. In the centre of the model, 3 duplets (red dot ellipses) initially gather 4 masses (yellow circles) taken from 4 different elementary physical entities into a single Mass-union (black ellipses). As long as these 3 duplets remain active, these 4 masses will share the same exact positions while computing. Central figure: while Mass-unions remain unsplit, all the masses gathered in each Mass-union behave as a single mass (orange circle). Right figure: the construction of the base topological structure inspired by the physics network. **(b) Association**. Association is based on the order of mass's placements (numbers) in the physics model. **(c) Modification**. In case the duplet between the mass 5 and 8 is inactivated by its Sensor, then the Mass-union splits into two Mass-unions: some masses previously tied are now separated. This information are used in the topological model to unsew in α_2 the two associated faces' corners (face's corner 2 with 11, and face's corner 5 with 8**). (d) and (e) Affectation step.** Two embedded geometry obtained from the topological structure in the course of the simulation. The first is the geometry of the topological model before the split, and the second is the geometry of the topologic model after the split. Only the edges that are free in α_2 (red large lines) are rendered in the geometric model.

The position given to each geometrical vertex can be the current value of the evolution function associated to any of the sewed face's corner forming the topological vertex.

To exemplify, in the case of our 2D simple model, in figure 3d before the split and 4e after the split, we choose the following geometrical embedding: each topological vertex becomes a geometrical vertex, each unsewed topological edge (without α_2 involution, in red) becomes a geometrical edge, etc. Hence, topological faces sewed with each other's are rendered as a single connected geometrical face.

When working with 3D volume models, the process is similar. We just have to take into account the third dimension's topological elements: the topological volumes. When several topological volumes are sewed, only the unsewed faces are rendered, so as to form a single large connected



(a) (b)

Figure 4: (a) Elementary physical tetrahedral entities with, in blue, constraint viscoelastic interactions. (b) The set of 7 duplets, that initially gathers 8 masses taken from 8 different elementary entities into a single Mass-union. The experimental model gathers 70x70x10 elementary physical entities.

component. This means that several sewed topological volumes in α_3 are rendered as a single geometrical volume. This is made very easy thanks to the topological model: the faces of such connected components are simply those which α_3 involutions are free. More complex geometrical embedding are possible, for example, one could also choose to embed another, possibly more complex, geometry, such as for example: adding geometrical vertices on the centre of each geometrical face; creating geometrical vertices in the centre of each topological volume; etc.

3 EXPERIMENTS AND COMPLEXITY

For the following experiments, the Physics network upstream is made of 70x70x10 hexahedral basic physical entities. Each entity, corresponding to the fully-split state, is made of 8 masses, and 16 interactions, forming a hexahedral physical entity (Figure 4a). Then, these entities are tied to each other's with Duplets (Figure 4b), so as to form a large 3D physics-based splittable block. To trigger duplet's inactivation, we employ distance Sensors mounted between two masses chosen in adjacent entities outside the Mass-union. In this article, from an experiment to another, only the physical parameters values, and the initial state, are modified.

On the topological side, the base topological model is made of 70x70x10 topological cubic volumes that are sewed with each other's to form a single large parallelepiped block (Figure 5).

The two animations shown on Figure 6a and Figure 6b were both achieved with the same exact physical, topological and geometrical models. The physical model stands for a slightly deformable matter, in which Sensors' thresholds are chosen non-homogeneous, so as to spread various cracking lines

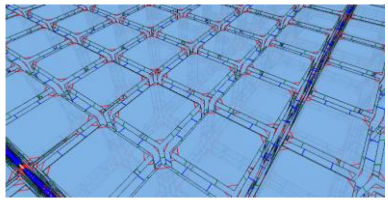


Figure 5: The 3D volume topological model is inspired from the building process employed in the physics' model: 70x70x10 topological cubic volumes are sewed to each other's.

in the matter. In between the two animations, only the initial state of the physical model differs. Depending on this initial state, cracks emerge and propagate at various places in the simulation.

The example shown on Figure 7 illustrates possibility of thoroughly different effects with the pipeline. In the physical model, the parameters are set to achieve a very deformable matter behaviour – like a thin sheet or soft body. Then, the Sensors' thresholds controlling Duplets inactivation are chosen inhomogeneous over the model: the distance thresholds are made smaller along a vertical line close to the centre of the model. Hence, we favour a chosen tear propagation line in the model.

The videos associated with this paper (*http://147.171.151.195:8080/fbsharing/LTWswZVk*) provide other examples illustrating further variability in the obtainable behaviours and renderings

In all these experiments, the physical model is run at 1050Hz: employing Splitting MAT leads to a fully stable complexity, no matter the occurring splits. The topo-geometrical part of the pipeline is run at 50 Hz, and we achieved interactive framerate for all our experiments on a standard PC.

The algorithmic complexity of the Modification step starts by analysing the received Splitting-Mat events. For each newly created sub-Mass-union, a local scan of orbit<α1 , α2> is performed to determine which volume's corners should be unsewed from each other's. The complexity is O(n*m*(n+2k)), with n the number of masses in the new Mass-union, m the number of faces incident on the volume's corner, and k the number of darts in each visited face. The number of split events to process on each step is the number of splits that occurred in the physics since the last execution of the topo-geometrical pipeline. Hence, in case many Mass-unions have split in the physics, the duration of the topological Modification step might be penalized. However, separating the physics from the geometry implies that this might only impact the stability of the visual framerate, but not at all the consistency of the physical simulation. As for it, the memory complexity of the topological model is

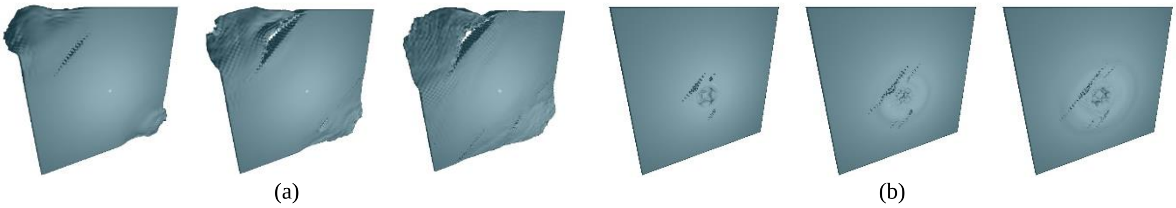


Figure 6: Emergence of tears in the matter. The same exact physical, topological and geometrical models are used for a) and b). Depending on the initial state of the physical model, various tears will emerge.

Figure 7: Tearing of a very deformable thin matter bloc. A tearing line is favoured by employing lower Sensor threshold in the centre of the physical model. (a) Direct representation of the physical model, by using a sphere on each mass. (b) Representation of the topological model, in which sewed and unsewed faces are rendered with distinct colours, allowing to pursue the evolving topological modifications. (c) Rendering of the finally obtained geometrical model.

constant throughout simulation, thanks to the use of the GMaps formalism: the number of darts is never changed, even when topology is modified.

4 CONCLUSIONS

This paper proposed new contributions to the today's stream of research that envisages employing explicit representations of the topological aspects to root the coating processes over geometry free physics-based methods. The general pipeline is made of cascading models: physical → topological → geometrical.

On the physical side, we extended the Splitting-MAT methodology, which enables the split to occur onto the material elements, to 3D volume models.

We then introduced downstream a topologico-geometrical pipeline adapted to this property of the upstream physical model. As compared to previous works on the 3-stages pipeline, which did not employ Splitting MAT, the split-on-the-masses property allows building rather simply a *base* topological model, and further enables a cunning handling of the topological transformations of this base structure during simulation, under control of the Physics.

The obtained results exhibit precision in both the dynamics and the visual (geometrical) aspects. Hence, the Splitting MAT-powered 3-stages pipeline, while being manageable, does not limit the richness of the desired dynamics (dynamics of the fracture and splits, propagation, etc.). Finally, a theoretical and practical measure of the complexity of the topologico-geometrical part shows that the pipeline competes with integrated approaches and qualifies for real time implementations.

In the future, we plan to creatively experiment with more diverse constructs. Indeed, handling physics (dynamics) / topology (spatial relations) /

geometry (final image) in 3 clearly separated cascading models introduces manageability in the modelling processes, and variability in the observation of the physically generated dynamics. Hence, besides employing the *base* topology and simple geometry, as presented in this article, the three models may indeed drastically differ in their structure, and in their complexity. We plan to build over this advantage to experiment with varied physical models upstream, and varied topological constructs and geometrical renderings for each of them: non-regular physical models, refined topologies, diverse geometrical embedding, etc. Rooting on formal approaches on both the physical and topological sides will ease such future explorations, we assume.

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