Question Selection for Interactive Program Synthesis

Ruyi Ji
Key Lab of High Confidence Software Technologies, Ministry of Education
Department of Computer Science and Technology, EECS, Peking University
Beijing, China
jiruyi910387714@pku.edu.cn

Jingjing Liang
Key Lab of High Confidence Software Technologies, Ministry of Education
Department of Computer Science and Technology, EECS, Peking University
Beijing, China
jingjingliang@pku.edu.cn

Yingfei Xiong*
Key Lab of High Confidence Software Technologies, Ministry of Education
Department of Computer Science and Technology, EECS, Peking University
Beijing, China
xiongyf@pku.edu.cn

Lu Zhang
Key Lab of High Confidence Software Technologies, Ministry of Education
Department of Computer Science and Technology, EECS, Peking University
Beijing, China
zhanglucs@pku.edu.cn

Zhenjiang Hu
Key Lab of High Confidence Software Technologies, Ministry of Education
Department of Computer Science and Technology, EECS, Peking University
Beijing, China
huzj@pku.edu.cn

Abstract

Interactive program synthesis aims to solve the ambiguity in specifications, and selecting the proper question to minimize the rounds of interactions is critical to the performance of interactive program synthesis. In this paper we address this question selection problem and propose two algorithms. *SampleSy* approximates a state-of-the-art strategy proposed for optimal decision tree and has a short response time to enable interaction. *EpsSy* further reduces the rounds of interactions by approximating *SampleSy* with a bounded error rate. To implement the two algorithms, we further propose *VSampler*, an approach to sampling programs from a probabilistic context-free grammar based on version space algebra. The evaluation shows the effectiveness of both algorithms.

CCS Concepts: • **Software and its engineering → Software notations and tools**; **General programming languages**.

Keywords: Interaction, Program Synthesis

ACM Reference Format:
Ruyi Ji, Jingjing Liang, Yingfei Xiong, Lu Zhang, and Zhenjiang Hu. 2020. Question Selection for Interactive Program Synthesis.

*Corresponding author

PLDI '20, June 15–20, 2020, London, UK

ACM ISBN 978-1-4503-7613-6/20/06…$15.00
https://doi.org/10.1145/3385412.3386025

In *Proceedings of the 41st ACM SIGPLAN International Conference on Programming Language Design and Implementation (PLDI '20), June 15–20, 2020, London, UK*. ACM, New York, NY, USA, 16 pages.
https://doi.org/10.1145/3385412.3386025

1 Introduction

Program synthesis [3, 20] finds a program meeting a given specification and has many applications [14, 15, 17, 35, 40, 46, 47, 50, 51]. A classic program synthesis problem assumes that the specification is complete, and the synthesizer only needs to find any program that meets the specification. However, in practice the specification is often incomplete, leading to ambiguity among many program candidates [36]. For example, in programming by example [18], it is common that the examples are few and many programs are consistent with the examples [38]. In program repair, it is also known that incorrect patches are often produced because of incomplete specification [34, 42].

A fundamental way to address the ambiguity is interactive program synthesis [30]. In interactive program synthesis, the system asks easy-to-answer questions to the user, and narrows down the program domain based on the answer from the user. The interaction ends when there is no ambiguity remaining. Compared to non-interactive program synthesis, interactive program synthesis adds the extra burden of answering questions to the user. Therefore, it is critical to minimize the number of questions needed to be asked.

As a matter of fact, the selection of questions has a great impact on the number of questions needed. Let us consider a program domain $\mathbb{P}_e$ defined by the following grammar:

$$S \coloneqq E \mid \text{if } E \leq E \text{ then } x \text{ else } y \qquad E \coloneqq 0 \mid x \mid y$$

There are nine semantically different programs in the domain, as follows.

(p_1) 0	(p_2) if $0 \le x$ then x else y	(p_3) if $0 \le y$ then x else y
(p_4) x	(p_5) if $x \le 0$ then x else y	(p_6) if $x \le y$ then x else y
(p_7) y	(p_8) if $y \le 0$ then x else y	(p_9) if $y \le x$ then x else y

To find out which program is the desired one, an interactive synthesis system may use input-output questions, i.e., in each round, the system selects an input, and asks the output of the input from the user. The selection of questions can significantly affect the number of questions. For example, when the target program is p_6, it is possible to finish synthesis with two questions: $(1, 2)$ and $(2, 1)$, where (a, b) represents input $x = a, y = b$. But if the synthesizer always chooses inputs from $\{(0, i) | i \ge 0\}$, the interaction will never finish since these inputs cannot distinguish p_6 from p_1.

As a result, it is important to study the *question selection* problem. Given a program domain $\mathbb{P}$ with a probability distribution φ on it, a domain $\mathbb{Q}$ of questions, and a sequence C of previous questions and answers, the *question selection* problem is to select the next question from $\mathbb{Q}$ such that the expected number of questions needed to remove ambiguity is minimized. As far as we are aware, the question selection problem has not been studied in the program synthesis community, and recent interactive program synthesis approaches all randomly select questions, leading to unnecessarily extra burden to the user.

In this paper we address the question selection problem for interactive program synthesis. Our basic idea is to adapt a state-of-the-art polynomial-time strategy for optimal decision tree problem, *minimax branch* [2, 11, 21], which selects the question where the worst user answer gives the best reduction of the program domain. In this example, $(-1, 1)$ is one best choice for the first question because it can exclude at least 5 programs whatever the answer is.

However, there are two main challenges in adapting *minimax branch*. First, applying this strategy requires to calculate for each program, each question, and each answer, whether the program is excluded or not. As a result, the time complexity is $O(|\mathbb{P}| \times |\mathbb{Q}| \times |\mathbb{A}|)$, where $|\mathbb{P}|$, $|\mathbb{Q}|$, and $|\mathbb{A}|$ are the sizes of the program domain, question domain, and answer domain, respectively. Even for this small example, $|\mathbb{P}| \times |\mathbb{Q}| \times |\mathbb{A}|$ is already 9×2^{96} (assuming x and y are 32-bit integers). Such a computation is almost impossible to be finished in a fast interactive session. Second, due to the complexity of program domains, *minimax branch* still results in too many questions for some practical problems. In our evaluation, an algorithm approximating *minimax branch* uses up to 18 questions, which involves too much burden to the user.

We propose two algorithms to address the two challenges. The first algorithm, *SampleSy*, approximates *minimax branch* using a Monte Carlo method: in each turn, *SampleSy* firstly samples a small set P of programs from the remaining program domain, and then applies *minimax branch* only to P. For example, if the samples are p_1, p_3, p_7, *SampleSy* still chooses $(-1, 1)$, which is the best input on distinguishing these three samples. In this way *SampleSy* removes the enumeration on the program domain. To further remove the enumerations on the question and answer domains, *SampleSy* generates constraints whose sizes only depend on $|P|$ and utilizes an SMT solver to solve them: whether there exists question q, such that at most t programs whose answers are the same. In this way, t is an upper bound of the worst-case program domain reduction for question q. By searching on t, *SampleSy* finds the best question q. Finally, since sampling P also requires a significant amount of time, *SampleSy* utilizes the intervals when the user answers the question to perform the sampling, and thus ensures a short response time. We show that *SampleSy* approximates *minimax branch* with a bounded probability.

The second algorithm, *EpsSy*, reduces the rounds of interactions by allowing bounded errors, i.e., potentially returning an incorrect program with a bounded probability. *EpsSy* reduces the rounds of interactions from two aspects. First, when a random sample from the program domain already ensures the bounded error rate, we stop the interaction process. For example, when the bounded error rate is 5%, if in the remaining program domain there is a program whose probability is larger than 95%, the interaction process will be stopped. Second, *EpsSy* further utilizes the fact that many modern synthesizers are able to accurately predict the desired program in many cases, and if a predicted program survives from a number of questions that are selected to challenge it, the predicted program is probably the correct program. For example, suppose the samples are p_1, p_3, p_7, a synthesizer predicts p_1 to be the target program, and *EpsSy* asks for $(-1, 1)$. If p_1 is incorrect, the probability for it to survive from this question will be about 1/3, since it performs differently from 2/3 samples. We show that the error rate of *EpsSy* is bounded, and is controlled by adjusting the number of samples and the number of challengeable questions.

To implement the two algorithms, we need a sampler to sample from the remaining program domain according to a distribution. In other words, the sampler not only finds programs that are consistent with the existing questions and answers, as a standard synthesizer does, but also needs to ensure the programs are drawn from a distribution. We introduce *VSampler* to perform this task. *VSampler* samples programs from a version space algebra (VSA) [49] according to a probabilistic context-free grammar (PCFG) [25]. PCFG is a standard way to model distributions over a set of programs represented by a CFG. VSA is a commonly-used data structure to represent a program domain reduced from input-output questions, which are the most common type of questions. Since the size of a program is also an important indicator of probability and PCFG does not directly support size-related distribution, *VSampler* also introduces a method

to annotate size on non-terminals such that size-related distributions can be modeled by PCFG.

We have evaluated our algorithm on 166 interactive synthesis problems for program repair and string manipulation. The results suggest that both algorithms significantly reduce the number of questions: a random strategy uses 38.5% and 13.9% more questions than *SampleSy*, and 54.4% and 35.0% more questions than *EpsSy*, respectively for program repair and string manipulation. Also, the error rate of *EpsSy* is small, where the overall error rate is 0.60%.

To sum up, this paper makes the following main contributions.

- *SampleSy* that selects questions with a short response time by approximating *minimax branch* with a bounded probability.
- *EpsSy* that further reduces the number of questions by allowing a bounded error.
- *VSampler* that samples programs from a VSA according to a PCFG, and can be used to implement *SampleSy* and *EpsSy*.
- An evaluation on a set of SyGuS problems showing the effectiveness of both *SampleSy* and *EpsSy*.

2 Question Selection Problem in Interactive Program Synthesis

In this section we define the question selection problem and introduce the *minimax branch* strategy.

2.1 Question Selection Problem

Definition 2.1 (Oracle Function)**.** Given a program domain $\mathbb{P}$, a question domain $\mathbb{Q}$, and an answer domain $\mathbb{A}$, an oracle function $\mathbb{D} : \mathbb{P} \to \mathbb{Q} \to \mathbb{A}$ associates a program p and a question q with an answer, denoted as $\mathbb{D}[p](q)$.

Obviously, interactive synthesis can only distinguish programs if the oracle function could distinguish them.

Definition 2.2 (Distinguishable Programs)**.** Two programs p_1 and p_2 are *distinguishable* w.r.t an oracle function $\mathbb{D}$ if and only if $\exists q \in \mathbb{Q}, \mathbb{D}[p_1](q) \neq \mathbb{D}[p_2](q)$, otherwise we say the two programs *indistinguishable*.

The goal of interactive program synthesis is to find a program p that is indistinguishable from a target program r. For the convenience of further description, we define the concept of *valid programs*, which are programs consistent with a set of question-answer pairs. In the following definitions, we implicitly assume the existence of a universal oracle function $\mathbb{D}$ for any program, question, and answer domains.

Definition 2.3 (Valid Programs)**.** Let $C \in (\mathbb{Q} \times \mathbb{A})^*$ be a sequence of question-answer pairs, and $\mathbb{P}$ be a set of programs. The valid programs with respect to C, denoted as $\mathbb{P}|_C$, are programs which are consistent with all these question-answer pairs, i.e., $\{p | p \in \mathbb{P}, \forall (q, a) \in C, \mathbb{D}[p](q) = a\}$.

Then we come to the interaction part. We first define what is considered as a question to ask.

Definition 2.4 (Question Selection Function)**.** Given a program domain $\mathbb{P}$, a question domain $\mathbb{Q}$, and an answer domain $\mathbb{A}$, a question selection function $QS : (\mathbb{Q} \times \mathbb{A})^* \mapsto \{\top\} \cup \mathbb{Q}$ is a function satisfying the following two conditions for any $C \in (\mathbb{Q} \times \mathbb{A})^*$ and $q^* = QS(C)$:

$$q^* = \top \iff \forall p_1, p_2 \in \mathbb{P}|_C, q \in \mathbb{Q}, \mathbb{D}[p_1](q) = \mathbb{D}[p_2](q) \quad (1)$$

$$q^* \neq \top \implies \exists p_1, p_2 \in \mathbb{P}|_C, \mathbb{D}[p_1](q^*) \neq \mathbb{D}[p_2](q^*) \quad (2)$$

Here, $\top$ represents the completed signal of the interaction. With a given question selection function QS, the interactive synthesis process will be carried out as follows (starting with $C = \emptyset$):

1. If $QS(C) = \top$, return any program in $\mathbb{P}|_C$.
2. Show $QS(C)$ to the developer and get the answer a.
3. Add $(QS(C), a)$ to C and return to step 1.

We use $len(QS, r)$ to represent the number of questions during the interaction.

Let φ be a priori probabilistic distribution on the program domain $\mathbb{P}$, which measures the likelihood for each program to be the target. The goal of the question selection problem is to find the optimal question selection function with respect to φ.

Definition 2.5 (Optimal Question Selection Function)**.** Let $\mathbb{P}$ be a program domain and φ be a distribution on $\mathbb{P}$. An optimal question selection function OQS is the question selection function that minimizes $\sum_{r \in \mathbb{P}} \varphi(r) len(OQS, r)$, i.e., the expected number of questions. For convenience, we call this expectation as the cost of a question selection function.

The optimal question selection problem, denoted as $\mathcal{OQS}$, is to implement an optimal question selection function. As can be found in the supplementary material [1], we show that $\mathcal{OQS}$ is polynomial-time transformable to the problem of constructing optimal decision tree [28], and vice versa. Since the optimal decision tree problem has been proved to be NP-hard, $\mathcal{OQS}$ is also NP-hard.

Theorem 2.6. *$\mathcal{OQS}$ is a NP-hard problem with respect to the size of $\mathbb{P}$, $\mathbb{Q}$ and $\mathbb{A}$.*

Due to space limit, we omit all proofs, and the details of the proofs can be found in the supplementary material [1].

2.2 A Greedy Strategy: Minimax Branch

As mentioned before, *minimax branch* is a polynomial-time strategy to approximate the optimal decision tree, and here we adapt it for optimal question selection. *minimax branch* is based on the following intuition: in order to minimize the expected number of questions, in each turn, the chosen question should disqualify as many programs as possible. For a program set P, define its *weighted size* $w(P)$ as $\sum_{p_0 \in P} \varphi(p_0)$, i.e., the probability for a random program to be in P. *minimax*

branch aims to minimize the worst-case weighted size in each turn.

Definition 2.7 (*minimax branch*)**.** *minimax branch* is a question selection function *minimax* defined as follows:

- If the interaction is finished, i.e., if $\forall p_1, p_2 \in \mathbb{P}|_C$, $\forall q \in \mathbb{Q}$, $\mathbb{D}[p_1](q) = \mathbb{D}[p_2](q)$, $minimax(C)$ will be $\top$.
- Otherwise, $minimax(C)$ is defined to be:

$$minimax(C) = \arg\min_{q \in \mathbb{Q}} \left(\max_{a \in \mathbb{A}} w\left(\mathbb{P}|_{C \cup \{(q,a)\}}\right) \right)$$

Based on the results from the optimal decision tree problem [2, 10, 11], *minimax branch* achieves the best approximation ratio in multiple variants of optimal decision tree problems and is a state-of-the-art strategy.

Theorem 2.8. *The minimax branch strategy approximates the optimal question selection function with an approximation ratio of* $O(\log^2 m)$*, where* m *is the size of the program domain.*

3 Optimized Algorithm I: *SampleSy*

In an interactive session, the response time is expected to be short, preferably in a couple of seconds. As mentioned before, implementing *minimax branch* requires to enumerate three large domains, which is almost impossible for an interactive algorithm.

In this section we introduce *SampleSy* that approximates *minimax branch* but has a short response time.

3.1 Overwiew

The basic idea of *SampleSy* is to sample a small number of valid problems, and then apply *minimax branch* only to these samples. In this way, the number of programs considered by *minimax branch* is sharply reduced. After that, instead of enumerating $\mathbb{Q}, \mathbb{A}$, *SampleSy* uses an SMT solver to find out the best question directly.

SampleSy is formed by three parallel processes:

- **Sampler**. A background process that continually samples programs from $\mathbb{P}|_C$.
- **Decider**. A background process that decides whether the termination condition is met, i.e., whether $\forall p_1, p_2 \in \mathbb{P}|_C, \forall q \in \mathbb{Q}, \mathbb{D}[p_1](q) = \mathbb{D}[p_2](q)$.
- **Controller**. The main process of *SampleSy*, a foreground process that deals with the interaction.

Algorithm 1 shows the pseudo code of the controller. In each turn (Lines 2-7), the controller firstly requires a set of samples P from the sampler $\mathcal{S}$ (Line 2), and then it applies *minimax branch* to find out the best question q^* for P (Line 3). After that, the controller shows q^* to the developer and waits for the corresponding answer a^* (Line 4). Finally it updates $\mathcal{S}$ and $\mathcal{D}$ with the new example (q^*, a^*) (Line 5): $\mathcal{S}$ will filter out the samples of which the answer on q^* is not a^*, and both $\mathcal{S}$ and $\mathcal{D}$ will update their internal data structure. The interaction repeats until the decider determines that the termination condition is met (Line 6).

Algorithm 1 The controller of *SampleSy*

Input: Instance $(\mathbb{P}, \mathbb{Q}, \mathbb{A}, \varphi)$ of $\mathcal{OQS}$, sampler $\mathcal{S}$ and decider $\mathcal{D}$.
Output: Synthesized program $p \in \mathbb{P}$

```
1: do
2:     P ← S.Samples
3:     q* ← Minimax(P, Q, A)
4:     a* ← Interact(q*)
5:     D.AddExample(q*, a*), S.AddExample(q*, a*)
6: while ¬ D.IsFinished
7: return Some program in S.Samples
```

In the following subsections, we shall go into the details of *SampleSy*.

3.2 Sampler $\mathcal{S}$

The sampler $\mathcal{S}$ is a program synthesizer that can repeatedly and independently sample programs from the remaining program domain $\mathbb{P}|_C$ according to distribution φ. More precisely, it samples programs from the conditional distribution $\varphi|_C$ on $\mathbb{P}|_C$ since other programs have been excluded:

$$\varphi|_C(p) = \Pr_{x \sim \varphi}[x = p | x \in \mathbb{P}|_C] = \varphi(p) w(\mathbb{P}|_C)^{-1}$$

In Section 5, we will show how to build an efficient sampler for distributions defined by probabilistic context-free grammars.

The sampler also needs to implement AddExample. When this method is called, the sampler checks whether the current samples are consistent with a new question-answer pair, and discard those that are inconsistent.

3.3 Decider $\mathcal{D}$

The decider $\mathcal{D}$ is a program synthesizer that can determine whether there are two distinguishable programs in $\mathbb{P}|_C$, i.e., the negation of the termination condition. Formally, $\mathcal{D}$ needs to determine whether there exists p_1, p_2 and q such that the following formula is satisfied.

$$\psi_{\text{unfin}}(p_1, p_2, q) \stackrel{\text{def}}{=\!=} (p_1, p_2 \in \mathbb{P}|_C) \wedge (\mathbb{D}[p_1](q) \neq \mathbb{D}[p_2](q))$$

where $p \in \mathbb{P}|_C \iff \left(\bigwedge_{(q',a) \in C} \mathbb{D}[p](q') = a\right) \wedge (p \in \mathbb{P})$.

In order to utilize SMT solvers, throughout this paper, we assume domain constraints $\exists p \in \mathbb{P}, \exists q \in \mathbb{Q}$ and the semantics function $\mathbb{D}$ can be translated into SMT queries and formulas respectively. Clearly, when the program domain $\mathbb{P}$ and the question domain $\mathbb{Q}$ are both finite, these assumptions always hold. Moreover, there have been lots of studies on effective encoding schemes [23, 37]: any of them can be used here.

Under these assumptions, ψ_{unfin} can be encoded into an SMT query, and $\mathcal{D}$ can be implemented by solving it with an SMT solver.

3.4 Question Selection

This subsection explains how to implement $\textsc{Minimax}(P, \mathbb{Q}, \mathbb{A})$ (Line 3) in Algorithm 1. According to *minimax branch* described in Section 2, the output of Minimax should be:

$$\textsc{Minimax0}(P, \mathbb{Q}, \mathbb{A}) = \arg\min_{q \in \mathbb{Q}} \left(\max_{a \in \mathbb{A}} \left| P|_{(q,a)} \right| \right)$$

where $P|_{(q,a)}$ denotes the samples in P which are consistent with question-answer pair (q, a). Note that since P is obtained by sampling, Minimax needs only to minimize the size of $P|_{(q,a)}$ rather than the weighted size.

There is a double loop on $\mathbb{Q}$ and $\mathbb{A}$ in Minimax0. To speed it up, we use an SMT solver to substitute the enumeration on $\mathbb{Q}$: We define formula $\psi_{\text{cost}}(q, t)$ to represent whether there are more than t programs perform the same on question q.

$$\psi_{\text{cost}}(q, t) \overset{\text{def}}{=\!=} \bigwedge_{a \in \mathbb{A}} \left(\left| P|_{(q,a)} \right| \leq t \right)$$

We assume $\mathbb{D}$ can be translated to an SMT formula and thus the whole formula are translatable. This formula tests whether there is a question with cost no more than a given threshold. Therefore, *SampleSy* can quickly find the best question by firstly calculating the smallest t that makes $\psi_{\text{cost}}(q, t)$ satisfiable and then finding a corresponding choice of q. There are various ways to find the smallest t. In our implementation, *SampleSy* searches on all possible values of t by binary search and checks the satisfiability by an SMT solver.

So far, the loop on $\mathbb{Q}$ has been eliminated. However, the size of the resulting formula $\psi_{\text{cost}}(q, t)$ is $O(|\mathbb{A}| \times |P|)$, which is too large for an SMT solver to cope with. To do further optimizations, we utilize the fact that the programs in P can only produce at most $|P|$ different answers on a given question. We construct the following formula $\psi'_{\text{cost}}(q, t)$ and thus reduce the length from $O(|\mathbb{A}| \times |P|)$ to $O(|P|^2)$.

$$\psi'_{\text{cost}}(q, t) = \bigwedge_{i=1}^{|P|} \left(\left(\sum_{j=i+1}^{|P|} [\mathbb{D}[p_i](q) = \mathbb{D}[p_j](q)] \right) \leq t \right)$$

To sum up, instead of enumerating $\mathbb{Q}$ and $\mathbb{A}$, *SampleSy* constructs the following formula and uses an SMT solver to find the best question quickly.

$$\textsc{Minimax}(P, \mathbb{Q}, \mathbb{A}) = \arg\min_{q \in \mathbb{Q}} t \ \text{ s. t. } \ \psi'_{\text{cost}}(q, t)$$

3.5 Parallelization

To further reduce the response time, we utilize the fact that the developer may take a while to answer the question. Therefore in *SampleSy*, Sampler $\mathcal{S}$ and Decider $\mathcal{D}$ are implemented as individual background processes so that they can utilize the pending time (Line 4 in Algorithm 1) to sample programs and evaluate the termination condition respectively.

If the sampler is too inefficient, there will not be enough samples for Minimax (Line 3) to find an efficient question. At this time, the developer can choose to wait more time for more samples, or use a potentially inefficient question. In Section 6, we shall demonstrate that our implementation is efficient enough for most real synthesis tasks.

So far, except $\textsc{Minimax}(P, \mathbb{Q}, \mathbb{A})$, all other calculations are processed in the background and do not affect the response time. Meanwhile, the time cost of $\textsc{Minimax}(P, \mathbb{Q}, \mathbb{A})$ can be controlled by limiting the size of P. In the implementation, we limit the response time to two seconds by setting an upper bound for $|P|$. We believe such a response time is small enough for an interactive algorithm.

3.6 Properties of *SampleSy*

In previous subsections, we discuss the optimizations in *SampleSy* and demonstrate that it is efficient on speed. In this subsection, we shall go back to the number of questions and show *SampleSy* performs well in this aspect. We first show that Minimax are equivalent to Minimax0.

Lemma 3.1. *For any $\mathcal{OQS}$ instance $(\mathbb{P}, \mathbb{Q}, \mathbb{A}, \varphi)$ and any $P \subseteq \mathbb{P}$, let q_0 and q_1 be the questions selected by Minimax0 and Minimax, respectively, then q_0 and q_1 have the same efficiency on P, i.e.:*

$$\max_{a \in \mathbb{A}} \left| P|_{(q_0,a)} \right| = \max_{a \in \mathbb{A}} \left| P|_{(q_1,a)} \right|$$

We then show that *SampleSy* well approximates *minimax branch*. Theorem 3.2 shows that for any $\epsilon > 0$, when $|P|$ is large enough, the question of *SampleSy* is almost surely a $1 + \epsilon$ approximation to that of *minimax branch*.

Theorem 3.2. *For any $\mathcal{OQS}$ instance $(\mathbb{P}, \mathbb{Q}, \mathbb{A}, \varphi)$ and any sequence of examples $C \in (\mathbb{Q} \times \mathbb{A})^*$, define the cost of an question cost(q) to be $\max_{a \in \mathbb{A}} w\left(\mathbb{P}|_{C \cup \{(q,a)\}}\right)$. Let P be a set of independent samples from $\varphi|_C$, q_1 and q_0 be the questions chosen by SampleSy and minimax branch respectively, then $\forall \epsilon > 0$:*

$$\Pr\left[cost(q_1) > (1+\epsilon) cost(q_0)\right] \leq 2d|\mathbb{Q}| \exp\left(-\frac{|P|\epsilon^2}{2d^2}\right)$$

where $d = \max_{q \in \mathbb{Q}} |\{\mathbb{D}[p](q) \mid p \in \mathbb{P}\}|$ which represents the maximum number of different answers on a single question.

Theorem 3.2 demonstrates that the probability of getting a bad question decreases exponentially when the number of samples increases, which can be summarized as the following corollary.

Corollary 3.3. *For any constant $\epsilon > 0$, we have:*

$$\Pr\left[cost(q_1) > (1+\epsilon) cost(q_0)\right] = O(\exp(-|P|))$$

4 Optimized Algorithm II: *EpsSy*

As mentioned before, the number of questions can be large, and we reduce them by allowing bounded errors. In this section we first introduce the problem *question selection with bounded error* and then introduces *EpsSy*, an algorithm built upon *SampleSy* but allowing bounded errors.

4.1 Question Selection with Bounded Error

Definition 4.1 (Unsafe Question Selection Function)**.** Given a program domain $\mathbb{P}$ defined on question domain $\mathbb{Q}$ and answer domain $\mathbb{A}$, an unsafe question selection function $US : (\mathbb{Q} \times \mathbb{A})^* \mapsto \mathbb{P} \cup \mathbb{Q}$ is a function satisfying the following two conditions for any $C \in (\mathbb{Q} \times \mathbb{A})^*$ and $x = US(C)$.

$$x \in \mathbb{P} \Longrightarrow \forall (q, a) \in C, \mathbb{D}[x](q) = a \tag{3}$$

$$x \in \mathbb{Q} \Longrightarrow \exists p_1, p_2 \in \mathbb{P}|_C, \mathbb{D}[p_1](x) \neq \mathbb{D}[p_2](x) \tag{4}$$

Similar to Definition 2.4, the interactive synthesis process can be determined by an unsafe question selection function US as follows (starting with $C = \emptyset$):

1. If $US(C) \in \mathbb{P}$, return $US(C)$.
2. Show $US(C)$ to the developer and get the answer a^*.
3. Add $(US(C), a^*)$ to C and return to Step 1.

Let r be the target program, we use $oup(US, r)$ to represent the program generated from the above process and $len(US, r)$ to represent the number of queries used by US.

Comparing Definition 4.1 with Definition 2.4, Definition 4.1 looses the first condition and allows the unsafe question selection function to "guess" a program at any time. Besides, the second condition remains unchanged, which guarantees the termination of an unsafe question selection function.

Then we define the concept of *error rate* and bound the error rate for unsafe question selection functions.

Definition 4.2 (Error Rate)**.** Let $\mathbb{P}$ be a program domain, φ be a distribution on the programs in $\mathbb{P}$, US be an unsafe question selection function. The error rate $e(US, \varphi)$ is the probability for US to return an incorrect program, i.e., a program distinguishable from r:

$$e(US, \varphi) = \Pr_{r \sim \varphi}[\exists q \in \mathbb{Q}, \mathbb{D}[oup(US, r)](q) \neq \mathbb{D}[r](q)]$$

Definition 4.3 (ϵ-Unsafe Question Selection Function)**.** Given program domain $\mathbb{P}$ and distribution φ on $\mathbb{P}$, an unsafe question selection function US is ϵ-unsafe iff $e(US, \varphi) \leq \epsilon$.

The optimal ϵ-unsafe question selection problem, denoted as $\mathcal{OUS}$, is to implement an optimal ϵ-unsafe question selection function which minimizes the expected number of questions on a prior distribution φ, i.e., $\sum_{r \in \mathbb{P}} \varphi(r) len(US, r)$.

4.2 Algorithm *EpsSy*

Now we come to *EpsSy*, an efficient algorithm for $\mathcal{OUS}$ which is built upon *SampleSy*. *EpsSy* is also formed by three parallel processes:

- **Sampler**. A background process that continually samples programs from $\mathbb{P}|_C$.
- **Recommender.** A background process that recommends the most probable program in $\mathbb{P}|_C$.
- **Controller**. The main process of *EpsSy*.

As we shall show later, the termination condition of *EpsSy* is simple enough to be calculated foreground. Therefore *EpsSy* does not require a background decider to check termination.

Algorithm 2 The controller of *EpsSy*

Input: Instance $(\mathbb{P}, \mathbb{Q}, \mathbb{A}, \varphi, \epsilon)$ of $\mathcal{OUS}$, a threshold f_ϵ, sampler $\mathcal{S}$ and recommender $\mathcal{R}$.
Output: Synthesized program $p \in \mathbb{P}$

```
1:  r ← R.Recommendation
2:  c ← 0
3:  do
4:      P ← S.Samples
5:      if ∃p* ∈ P, OccurNumber(P, p*) ≥ (1 − ε/2)|P| then
6:          return p*
7:      end if
8:      q*, v ← GetChallengeableQuery(r, P, Q, A)
9:      a* ← Interact(q*)
10:     S.AddExample(q*, a*), R.AddExample(q*, a*)
11:     if D[r](q*) = a* then
12:         c ← c + v
13:     else
14:         c ← 0, r ← R.Recommendation
15:     end if
16: while c < f_ε
17: return r
```

Algorithm 2 shows the pseudo code of the controller. *EpsSy* maintains a recommendation r (Line 1) and a confidence value c to it (Line 2). In each turn (Lines 3 to 16), *EpsSy* firstly requires a set P of samples from sampler $\mathcal{S}$ and uses an SMT solver to check whether P contains at least $(1-\epsilon/2)|P|$ indistinguishable samples (Line 5). If P does, *EpsSy* returns one of them as the answer directly (Line 6). Otherwise, *EpsSy* uses a question q^* to challenge the recommendation (Line 8), and v is the difficulty of this question. After the developer feeds back the corresponding answer (Line 9), *EpsSy* updates $\mathcal{S}, \mathcal{R}$ (Line 10) and check whether r survives from this question (Line 11). If so, the confidence will be increased by the difficulty (Line 12). Otherwise *EpsSy* will recalculate r and clear the confidence (Line 14). *EpsSy* returns r as the answer once the confidence reaches a threshold f_ϵ (Line 16).

Now we go into the details of *EpsSy*.

4.2.1 Recommender $\mathcal{R}$.

EpsSy utilizes the fact that a modern synthesizer is able to accurately infer the program desired by the user. *EpsSy* relies $\mathcal{R}$ to recommend likely programs and uses questions to verify them.

Note that the recommender can be any synthesizer that is able to synthesize a program consistent with the answers. Moreover, as we shall show later, the output of this synthesizer does not affect the bound on the error rate. Despite this, the more accurate the recommendation is, the earlier the target program will be found, and the fewer questions will

be asked. To reduce the number of questions, *EpsSy* expects $\mathcal{R}$ to be accurate in recommending the desired program.

4.2.2 Termination Condition. Compared with *SampleSy*, *EpsSy* uses two different termination conditions:

- *A certain proportion of the samples is indistinguishable* (Line 5). The samples reflect the distribution on $\mathbb{P}|_C$: if most of the samples are indistinguishable, with high probability, most of the programs in $\mathbb{P}|_C$ will be indistinguishable, too.
- *The confidence reaches a threshold* (Line 16). If the recommendation r is incorrect, it has non-zero probability to be excluded in each turn. Therefore, if a recommendation survives from many questions, with high probability, it will be the correct answer.

Both of these two conditions can be evaluated efficiently: The first condition can be verified by checking distinguishability for $O(|P|)$ pairs of samples, which can be efficiently done by an SMT solver with the following formula:

$$\psi_{dist}(p_1, p_2) = \exists q \in \mathbb{Q}, \mathbb{D}[p_1](q) \neq \mathbb{D}[p_2](q)$$

and the second condition is just a comparison. Therefore *EpsSy* puts them in the foreground. In Subsection 4.4, we shall show the error rate of *EpsSy* is bounded by these two conditions.

4.3 Question Selection

This subsection is about GetChallengeableQuestion (Line 8) in Algorithm 2. This function aims to find a question which has a high probability of excluding an incorrect recommendation. To ensure it, *EpsSy* tries to locate a question where the proportion of the sample programs returning an answer different from r is at least w. In this way, if r is not the desired program, the developer's answer will have around w probability to exclude r. More formally, we introduce a formula $\psi_{\text{good}}(q, w)$ as follows ($P\backslash r$ denotes the programs that are distinguishable from r in P, and can be obtained by checking the distinguishability between programs in P and r by ψ_{dist} since both P and r are known):

$$\psi_{\text{good}}[r](q, w) \overset{\text{def}}{=\!=} \left(\sum_{p \in P\backslash r} [\mathbb{D}[p](q) = \mathbb{D}[r](q)] \right) \leq (1 - w)|P|$$

To faster exclude incorrect recommendations, the question should satisfy $\psi_{\text{good}}[r](q, w)$ with a larger w; to minimize the size of $|\mathbb{P}|_C|$, the question should instead satisfy $\psi_{cost}(q, t)$ with a small t. *EpsSy* makes a tradeoff between these two aspects as shown in Algorithm 3. If there exists a question which is good (Line 1), *EpsSy* will find the question with the smallest cost among all good questions, and return it with difficulty $v = 1$ (Lines 2, 3). Otherwise, *EpsSy* does the same as *SampleSy* and return with difficulty $v = 0$ (Lines 5, 6). Intuitively, for an incorrect recommendation to be returned, it has to survive on f_ϵ good questions, of which the probability is only around $(1 - w)^{f_\epsilon}$: thus the error rate is bounded.

Algorithm 3 The question selection in *EpsSy*

Input: Recommendation r, samples P, question domain $\mathbb{Q}$ and answer domain $\mathbb{A}$.
Output: A question q^* and the corresponding difficulty v.

```
1: if ψ_good[r](q, w) is satisfiable then
2:     q* ← arg min_q t s.t. ψ_good[r](q, w) ∧ ψ'_cost(q, t)
3:     return q*, 1
4: else
5:     q* ← arg min_q t s.t. ψ'_cost(q, t)
6:     return q*, 0
7: end if
```

Example 4.4. Assume the program domain is $\mathbb{P}_e$ (the program domain discussed in Section 1), the samples P are p_1, p_2, p_4,p_5,p_7,p_8 and the recommendation is p_7, then:

- When $w = 0.5$, one best question is input $(-1, 1)$ on which p_7 performs differently with p_1, p_4, p_5, accounting for 60% in $P\backslash r$, and $(-1, 1)$ can exclude 3 samples in the worst case.
- When $w = 0.9$, one best question is input $(0, -1)$ on which p_7 performs differently with all samples in $P\backslash r$, and $(0, -1)$ can only exclude 1 sample in the worst case.

A delicate point is how to determine the value of w. To achieve a certain error rate, the larger w is, the smaller f_ϵ will be, and thus the fewer questions required by *EpsSy* will be. On the other hand, the larger w is, the fewer questions on which $\psi_{\text{good}}[r](q, w)$ is satisfiable will be: it may affect the performance of *EpsSy* on excluding samples, as shown in Example 4.4. Lemma 4.5 shows that $w = 1/2$ is a threshold of the satisfiability of $\psi_{\text{good}}[r](q, w)$.

Lemma 4.5. *For any program set P and constant w, define $B(P) \subseteq P$ as the set of programs which makes $\psi_{good}[p](q, w)$ unsatisfiable, i.e., $B(P) = \{p \in P | \forall q \in \mathbb{Q}, \neg\psi_{good}[p](q, w)\}$:*

- *$w \leq 1/2 \Longrightarrow \forall P$, programs in $B(P)$ are indistinguishable from each other.*
- *$w > 1/2 \Longrightarrow \forall t > 0, \exists P$ s.t. $B(P)$ contains at least t programs which are distinguishable from each other.*

Lemma 4.5 shows that if $w \leq \frac{1}{2}$, whatever P is, the probability that $\psi_{\text{good}}[r](q, w)$ is unsatisfiable is always small; but once w is larger than $\frac{1}{2}$, this probability can be extremely large. Therefore, in our implementation, *EpsSy* sets w to 1/2.

4.4 Properties of *EpsSy*

Theorem 4.6 demonstrates the most important property of *EpsSy*: it can reach an arbitrarily small error rate with logarithmic level f_ϵ.

Theorem 4.6. *For an $\mathcal{OUS}$ instance $(\mathbb{P}, \mathbb{Q}, \mathbb{A}, \varphi, \epsilon)$, let n be a lower bound of the number of samples in each turn, β be an*

upper bound of questions required by EpsSy, then:

$$\forall n > \max\left(18\ln\left(\frac{2\beta|\mathbb{Q}|}{\epsilon}\right), \frac{16\ln 2}{\epsilon^2} + \frac{8}{\epsilon^2}\ln\left(\frac{1}{\epsilon}\right)\right),$$
$$\forall f_\epsilon > \log_{3/2}\left(\frac{2\beta}{\epsilon}\right), e(EpsSy, p) \le \epsilon$$

5 Sampler

In this section, we shall introduce a sampler *VSampler* which uses *version space algebra* to represent the set of valid programs $\mathbb{P}|_C$ and can efficiently sample programs from distributions defined by *probabilistic context-free grammars*. *VSampler* supports input-output questions, one of the most commonly used type of questions.

5.1 Preliminaries

We start from *Context-free Grammar (CFG)*.

Definition 5.1 (Context-Free Grammar)**.** *Context-free grammar (CFG)* G is a 4-tuple (N, Σ, R, S) where N is a set of non-terminal symbols, Σ is a set of terminal symbols, R is a subset of $N \times (N \cup \Sigma)^*$ which represents a set of rules and S is the start symbol. A rule (α, β) (or denoted as $\alpha := \beta$) represents the nonterminal symbol α can produce sequence β. G contains all the sequences comprised of only terminal symbols that can be produced by the start symbol S directly or indirectly.

Throughout this section, we assume the grammar is always unambiguous, i.e., for any sequence contained in the grammar, there exists a unique leftmost derivation from the start symbol to the sequence.

VSampler assumes the program space can be represented by a *version space algebra (VSA)*. VSA is a subset of CFG where only the following three kinds of rules are allowed and each non-terminal can only appear in the left once:

$$\begin{array}{ll} s := p_1|p_2|\ldots|p_k & s \in N, p_i \in \Sigma^* \\ s := s_1|s_2|\ldots|s_k & s \in N, s_i \in N \\ s := F(s_1, \ldots, s_k) & s \in N, F \in \Sigma, s_i \in N \end{array}$$

Note that the first two kinds of rules can be split into k different rules $s := p_i$ ($s := s_i$) in CFG.

Example 5.2. The program domain $\mathbb{P}_e$ can be represented by the following VSA:

$$S := E|S_1 \quad S_1 := \text{if } (E, E) \quad E := 0|x|y$$

where if (E, E) is an abbreviation for if $(E \le E)$ then x else y and the start symbol is S.

Now we come to the PCFG model which is an extension of CFG by assigning probabilities to the rules.

Definition 5.3 (Probabilistic Context-free Grammar)**.** A probabilistic context-free grammar G is a context-free grammar (CFG) (N, Σ, R, S) combined with a function $\gamma : R \mapsto \mathbb{R}^+$, which satisfies $\forall \alpha \in V, \sum_{(\alpha,\beta)\in R} \gamma(\alpha, \beta) = 1$.

The PCFG model uses $\gamma(\alpha, \beta)$ to represent the probability of choosing rule (α, β) to expand a non-terminal symbol α. Therefore, for a program which is produced by rule (α_1, β_1), $\ldots$, (α_n, β_n), its probability is equal to $\prod_{i=1}^n \gamma(\alpha_i, \beta_i)$.

Since VSA is a subset of CFG, the PCFG model can also be defined on a VSA.

Example 5.4. The following PCFG defines a uniform distribution on the VSA in Example 5.2.

$S := E$	1/4	$S := S_1$	3/4	$S_1 := \text{if } (E, E)$	1
$E := 0$	1/3	$E := x$	1/3	$E := y$	1/3

Rule "$S := E$" and "$E := 0$" are used to obtain "0", therefore the probability of "0" is equal to $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. For program "if $x \le x$ then x else y", "$S := S_1$" and "$S_1 := \text{if } (E, E)$" are used once while "$E := x$âĂIJ is used twice, therefore its probability is $\frac{3}{4} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{12}$.

In program synthesis frameworks such as *FlashMeta* [17, 29, 41], a VSA is transformed to represent a subset of the program domain that satisfies the specification. Such a transformation is performed by annotating the non-terminals in a VSA with constraints that the programs expanded from the non-terminal must satisfy. When a non-terminal represents an expression and the specification is given as input-output examples, a common annotation is the return value of the expression. More concretely, given a VSA $G = (N, \Sigma, R, S)$ and a set of input-output examples $\{I_i \to O_i\}_{i=1}^n$, a new VSA G' is constructed to represent programs in G which are consistent with all examples. In the new G', each symbol has the form $\langle s, o_1, \ldots, o_n\rangle$ which represents a set of programs which (1) can be produced by non-terminal symbol s in G, and (2) output o_i on the ith input I_i. As a result, symbol $\langle S, O_1, \ldots, O_n\rangle$ represents a set of programs consistent with all examples. The rules in G' are constructed according to the rules in G and the semantics of the operators: every rule in G' can be mapped back to some rule in G. Example 5.5 demonstrates this process.

Example 5.5. The following grammar is the transformed version from the VSA in Example 5.2 based on input-output pair $(0, 1) \to 0$. We take $\langle S, 0\rangle$ as an example to show how the rules are constructed. $\langle S, 0\rangle$ represents programs which are expanded from S and output 0 on input $(0, 1)$, and there are two ways to get such programs: either through sub-rule $S \to E$ or through sub-rule $S \to S_1$. Both cases require a program which is expanded from the right symbol and still outputs 0. Therefore the rule of $\langle S, 0\rangle$ should be $\langle E, 0\rangle \mid \langle S_1, 0\rangle$.

$$\langle S, 0\rangle := \langle E, 0\rangle \mid \langle S_1, 0\rangle \qquad \langle E, 0\rangle := x \mid 0 \qquad \langle E, 1\rangle := y$$
$$\langle S_1, 0\rangle := \text{if } (\langle E, 0\rangle, \langle E, 0\rangle) \mid \text{if } (\langle E, 0\rangle, \langle E, 1\rangle) \mid \text{if } (\langle E, 1\rangle, \langle E, 1\rangle)$$

Strictly speaking, this grammar is not a VSA, but it can be converted to a VSA by adding auxiliary non-terminal symbols to split the last rule.

5.2 Sampling from VSA

Given program domain $\mathbb{P}$ defined on VSA G, examples C and a distribution φ over $\mathbb{P}$ defined by a PCFG, *VSampler* firstly transforms G to the VSA G' representing $\mathbb{P}|_C$ as in existing approaches [17, 29, 41], and then samples programs from G' according to the distribution $\varphi|_C$.

VSampler uses two functions GetPr(N) and Sample(N) to perform sampling. Given a PCFG $(V, \Sigma, R, S, \gamma)$, the definitions of these two functions are shown in Figure 1. For each non-terminal symbol N, *VSampler* firstly uses GetPr to preprocess the sum of the probabilities of the programs in N. Utilizing those probabilities, function Sample samples programs according to $\varphi|_C$ by recursively sampling the subprograms.

Example 5.6. Assume *VSampler* is going to sample programs from VSA in Example 5.5 and the PCFG in Example 5.4. Firstly, it uses GetPr to calculate the probability for each non-terminal symbol. and the results are:

$$\textsc{GetPr}(\langle E,0\rangle) = 2/3 \qquad \textsc{GetPr}(\langle E,1\rangle) = 1/3$$
$$\textsc{GetPr}(\langle S_1,0\rangle) = 7/9 \qquad \textsc{GetPr}(\langle S,0\rangle) = 3/4$$

Now consider the probability for"if $x \le y$ then x else y" to be sampled. Starting from $\langle S,0\rangle$, with probability 7/9, Sample selects programs from $\langle S_1,0\rangle$. Then, with probability 2/7, it recurses into "if $(\langle E,0\rangle, \langle E,1\rangle)$": The first parameter has 1/2 chance to be x while the second parameter must be y. In conclusion, the probability of "if $x \le y$ then x else y" is $7/9 \times 2/7 \times 1/2 = 1/9$, which is consistent with $\varphi|_C$.

5.3 Properties of *VSampler*

Theorem 5.7 demonstrates the correctness of *VSampler* on finite program domains.

Theorem 5.7. *For a program domain $\mathbb{P}$, an example sequence C and a distribution φ defined by a PCFG, let G be an acyclic VSA that represents $\mathbb{P}|_C$, then Sample(S) is subject to the conditional distribution $\varphi|_C$, where S is the start symbol of V.*

Besides the correctness, *VSampler* is also efficient on the time cost. Let m be the number of rules in the VSA, k_0 be the maximum k among all rules and s_0 be an upper bound of the programs' size, then the time complexities of GetPr and Sample are $O(mk_0)$ and $O(s_0k_0)$, respectively. So the time complexity for *VSampler* to generate t samples is $O(mk_0 + ts_0k_0)$. Note that constructing such a VSA requires at least $\Omega(mk_0)$ time, which means performing sampling is not the bottleneck of *VSampler*, especially when the number of samples is not large.

5.4 Representing Size-Related Distribution

The success of enumeration based synthesizers [3, 4] has shown that the size of a program is an important indicator of finding the most probable program. However, the PCFG model cannot encode any size-related distribution, since it is based on a context-free grammar. *VSampler* resolves this problem by constructing an auxiliary CFG which encodes size information into non-terminals.

Definition 5.8 (Auxiliary CFG)**.** Given a CFG (V, Σ, R, S) and a size limit n, its auxiliary CFG is a context-free grammar $((V \times [n]) \cup \{S'\}, \Sigma, R', S')$, where R' is defined as:

- For any $(\alpha, \beta) \in R$ with k non-terminals in β, for any positive integers $s_1, \ldots, s_k \in [n]$, $(\langle \alpha, 1 + \sum_i s_i\rangle, \beta')$ is added to R', where β' is constructed from β by replacing the ith non-terminal v_i with $\langle v_i, s_i\rangle$.
- For any $s \in [n]$, $(S', \langle S, s\rangle)$ is added to R'.

For any CFG G and size limit n, its auxiliary CFG G' contains exactly all programs in G with size at most n. Thus PCFGs on G' can represent size-related distributions on G.

Example 5.9. The following PCFG represents a distribution on the VSA in Example 5.2, where the probability of a program is inversely proportional to its size:

$S' := \langle S,1\rangle$	1/2	$S' := \langle S,3\rangle$	1/2	$\langle S,1\rangle := \langle E,1\rangle$	1
$\langle S,3\rangle := \langle S_1,3\rangle$	1	$\langle S_1,3\rangle :=$ if $(\langle E,1\rangle, \langle E,1\rangle)$			1
$\langle E,1\rangle := 0$	1/3	$\langle E,1\rangle := x$	1/3	$\langle E,1\rangle := y$	1/3

In this model, the probability of "0" is $\frac{1}{2} \times 1 \times \frac{1}{3} = \frac{1}{6}$ and the probability of "if $x \le x$ then x else y" is $\frac{1}{2} \times 1^2 \times \frac{1}{9} = \frac{1}{18}$.

6 Evaluation

To evaluate *SampleSy* and *EpsSy*, we report several experiments designed to answer the following research questions:

- **RQ1**: How do *SampleSy* and *EpsSy* compare against existing strategies?
- **RQ2**: How does the prior distribution affect the performance of *SampleSy*?
- **RQ3**: How does the number of the samples affect the performance of *SampleSy* and *EpsSy*?
- **RQ4**: How does the value of f_ϵ affect the performance of *EpsSy*?

6.1 Implementation

We implemented *SampleSy* and *EpsSy* in C++ and used Z3 [13] as the underlying SMT solver for them. Our implementation uses the SyGuS format to specify the synthesis task, and all the questions are in the form of input-output examples. Our implementation is open source and can be found in the supplementary material [1].

For *VSampler*, we implemented the algorithm in FlashMeta [41] to construct VSAs. For *SampleSy*, we used *Second Order Solver* [37], a state-of-the-art solver-based synthesizer, to implement the decider. For *EpsSy*, we chose *Euphony* [31], a synthesizer utilizing learned probabilistic models to find the best program, to be the recommender; for the benchmarks which are not supported by *Euphony*, we took *Eusolver* [4], an efficient enumerative based synthesizer, instead.

$$\textsc{GetPr}(s) = \begin{cases} \sum_{i=1}^{k} \gamma(\sigma(s, p_i)) \\ \sum_{i=1}^{k} (\gamma(\sigma(s, s_i))\textsc{GetPr}(s_i)) \\ \gamma(\sigma(s, F)) \prod_{i=1}^{k} \textsc{GetPr}(s_i) \end{cases} \qquad \textsc{Sample}(s) = \begin{cases} p_i, i \propto \gamma(\sigma(s, p_i)) & s := p_1 | \ldots | p_k \\ \textsc{Sample}(s_i), i \propto (\gamma(\sigma(s, s_i))\textsc{GetPr}(s_i)) & s := s_1 | \ldots | s_k \\ F(\textsc{Sample}(s_1), \ldots, \textsc{Sample}(s_k)) & s := F(s_1, \ldots, s_k) \end{cases}$$

Figure 1. The definitions of function GetPr and Sample used by *VSampler*. $i \propto f(i)$ represents i is sampled from a distribution where $\Pr[i = a]$ is proportional to $f(a)$, and σ is a function maps annotated rules in G' back to the origin rules in G.

6.2 Compared Approach and Configuration

We implemented the random strategy used by Mayer et al. [36], denoted as *RandomSy*, as the baseline for RQ1. In each turn, *RandomSy* repeatedly selects a question q from $\mathbb{Q}$ randomly, until there are two remaining programs performing differently on q, i.e., $\exists p_1, p_2 \in \mathbb{P}|_C, \mathbb{D}[p_1](q) \neq \mathbb{D}[p_2](q)$. Such a formula can be evaluated by the decider described in Subsection 3.3, and in our implementation, *RandomSy* and *SampleSy* share the same decider.

For all strategies, we used a simulator to simulate the human-computer interaction: For each question (Line 4 in Algorithm 1 and Line 9 in Algorithm 2), the simulator firstly delays 1 minute to simulate the time cost for the developer to calculate the answer, and then sends the correct answer to the controller. Since this paper focuses on selecting the best question, we ignored other potential problems during the human-computer interaction, e.g., the user may make mistakes, for simplicity. Designing solutions for these problems is orthogonal to question selection and is future work. Besides, the response time, i.e., the time cost of Minimax in Algorithm 1 and GetChallengableQuery in Algorithm 2, are limited to 2 seconds: when the number of samples is too large, starting from a small subset, we gradually extend the set until the time is used up.

For *EpsSy*, we set the threshold f_ϵ to 5 by default. For *SampleSy* and *EpsSy*, we constructed a size-related distribution φ_s as the default prior distribution: $\varphi_s(p) = (S \times n_{\text{size}(p)})^{-1}$, where S is the maximum size of programs in $\mathbb{P}$ and n_i is the number of programs with the size i in $\mathbb{P}$. φ_s has two advantages: (1) The size of p is subject to a uniform distribution, i.e., φ_s does not have any preference on size, which lets the samples better represent the whole space. (2) For a program space defined by a grammar, n_i increases exponentially when i increases. Therefore the smaller the size is, the more probable a program will be chosen.

All of the following experiments were conducted on Intel Core i7-8700 3.2 GHz 6-Core Processor with 32GB of RAM. Our experiment data are available in the supplementary material [1].

6.3 Benchmarks and Experiments

We constructed two datasets *Repair* and *String* corresponding to two promising applications of interactive synthesis: program repair and string manipulation (e.g., data wrangling tasks in spreadsheets):

Repair. This dataset is based on the *Program Repair* track in SyGuS competition which contains 18 benchmarks extracted from the program repair process of real-word Java bugs. In each benchmark, a program needs to be synthesized from a given grammar G to satisfy a set of input-output examples. We used the following way to construct interactive synthesis benchmarks: (i) The program domain $\mathbb{P}$ is defined by G plus a depth limitation. (ii) The question domain $\mathbb{Q}$ is defined by the parameters of the target program, e.g. if the program takes two integers as the input, then $\mathbb{Q} = \mathbb{Z} \times \mathbb{Z}$ where $\mathbb{Z}$ represents the set of integers. (iii) The target program r is a program satisfying the input-output examples. Please note that the input-output examples are not available to the interactive synthesis systems.

String. This dataset is based on the string benchmarks collected by Lee et al. [31], which contains 108 string-related benchmarks in SyGuS, 37 questions by spreadsheet users in StackOverflow, and 60 articles about Excel programming in Exceljet. All the benchmarks correspond to common data manipulation tasks in spreadsheet and a set of input-output examples are given for each benchmark. We converted those benchmarks into interactive synthesis tasks in the following way: (i) The program domain $\mathbb{P}$ is defined by the grammar used in *FlashFill* [17] plus a depth limitation. We did not use the original grammars in the benchmarks because they contain int/string conversions that are not supported by *FlashMeta* framework, which is used to implement *VSampler*. (ii) The question domain $\mathbb{Q}$ contains all inputs in given examples. The maximum number of examples per benchmark is 400 and the average number is 45.8. We did not include inputs beyond the examples, because the tasks in string benchmarks are all data wrangling tasks, where it is not necessary to process data outside the benchmark. (iii) The target program r is a program satisfying all input-output examples.

While constructing benchmarks, a depth limit d was used to make the program space finite, and for all benchmarks, we set d to the maximum value that can be dealt by the client synthesizers within 1 minute: *FlashFill* in the sampler, *Second-order Solver* in the decider, *EuSolver* and *Euphony* in the recommender. Besides, we filter out all benchmarks which cannot be solved by those solvers even with smallest possible d. Table 1 lists the information of the final datasets.

Based on the two datasets, we conducted four sets of experiments to answer the four research questions. In all the experiments, we repeated each single execution 5 times and

Table 1. The overview of *Repair* and *String*

Name	#Benchmarks	Average $\lvert\mathbb{P}\rvert$	Maximum $\lvert\mathbb{P}\rvert$
Repair	16	2.4×10^8	3.8×10^{14}
String	150	4.0×10^{25}	5.3×10^{91}

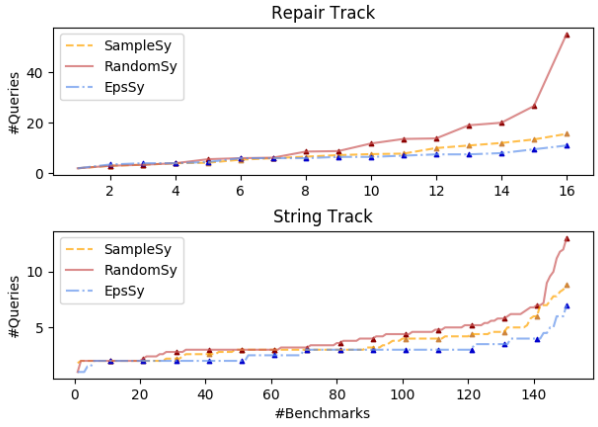
The second column lists the number of benchmarks in each set. The third column shows the geometric mean of $\lvert\mathbb{P}\rvert$ in each set while the forth column shows the largest one.

measured the average value, since all the approaches have randomness.

6.4 Exp 1: Comparison of Approaches (RQ1)

Procedure. In this experiment, we ran *SampleSy*, *RandomSy* and *EpsSy* on all benchmarks in two datasets until a program is synthesized and measured the number of questions required by them.

Results. Figure 2 shows the results of this experiment. From this figure, we make the following observations.



For each approach, we sort the benchmarks in the increasing order of the number of questions and plot the ith benchmark as a point (i, y_i) where y_i is the average number of questions on it.

Figure 2. The results of experiment 1

SampleSy. On average, *RandomSy* requires 38.5% and 13.9% more questions than *SampleSy* in *Repair* and *String* dataset respectively. Besides, Figure 2 shows that the performance gap between *SampleSy* and *RandomSy* increases as the difficulty of the benchmark rises: On the hardest 30% benchmarks, *RandomSy* requires 117% more questions than *SampleSy* in *Repair* and 24.8% more questions in *String*.

EpsSy. The overall error rate of *EpsSy* is 0.60%. On average, *RandomSy* requires 54.4% and 35.0% more quires than *EpsSy* in the two datasets. When only the hardest 30% benchmarks are considered, those ratios rise to 269% and 84.6%, respectively. Comparing to *SampleSy*, *EpsSy* can further reduce the number of questions even with a small error rate.

The tendency that *SampleSy* and *EpsSy* perform better on hard tasks is caused by the internal greedy strategy: *minimax branch* aims to improve the quality of each question, and its benefits accumulate during the interaction. Therefore, the more the questions are, the longer the interaction will be, and thus the more significant the accumulated advantages of *SampleSy* and *EpsSy* will be.

6.5 Exp 2: Comparison of Prior Distributions (RQ2)

Procedure. In this experiment, we tested the sensitivity of our approaches on the choice of prior distribution. As discussed in Section 6.2, *SampleSy* and *EpsSy* take $\varphi_s(p) = (S \times n_{\text{size}(p)})^{-1}$ as the prior distribution by default, where S is the maximum size of programs in $\mathbb{P}$ and n_i is the number of programs with size i in $\mathbb{P}$. Besides, we further considered the following distributions:

- *Enhanced* φ_s, in which we manually enhanced the accuracy of φ_s. At each sampling, with a probability of 0.1, the sampler directly returns the target program, and samples a program according to φ_s otherwise.
- *Weakened* φ_s, in which we manually weakened the accuracy of φ_s. At each sampling, the sampler firstly samples a program p according to φ_s. If p is indistinguishable from the target program, with a probability of 0.5, the sampler resamples a program, and returns p otherwise.
- *Uniform* Distribution φ_u, in which the probability of all programs are equal.

Instead of sampling programs from a given distribution, many state-of-the-art synthesizers find the top-k programs according to a given ranking function [31, 41]. Therefore, we also explored the possibility of directly using these synthesizers as samplers: we evaluated a strategy, denoted as *Minimal*, in which the sampler enumerates programs in the increasing order of the size instead of truly sampling.

For each distribution mentioned above, we measured the average number of questions required by *SampleSy* and *EpsSy* on both datasets.

Results. The results are summarized in Table 2. In general, the effectiveness ranking of these distributions is: Enhanced φ_s > Default φ_s > Weakened φ_s > Uniform $\varphi_u \approx$ Minimal, which demonstrates that the accuracy of the prior distribution does have a positive correlation to the performance of *SampleSy* and *EpsSy*. On the other hand, the impact is not significant: even when the distribution is Uniform φ_u or Minimal, *SampleSy* and *EpsSy* still perform much better than *RandomSy*. This result suggests that even if the distribution cannot be precisely determined, our algorithms can still perform well by either using a coarse distribution like the uniform distribution or directly using off-the-shelf synthesizers as samplers.

One surprising result is that in String, *SampleSy* with Uniform φ_u performs better than φ_s. A possible reason is that in the last few rounds, most incorrect programs (programs distinguishable from the target) have been excluded, and thus the probability of sampling remaining incorrect

Table 2. The average number of required questions for *SampleSy* and *EpsSy* with each considered distribution on each dataset.

Distribution	*SampleSy*			*EpsSy*		
	Repair	String	Combined	Repair	String	Combined
Weakened φ_s	8.100	3.511	3.947	6.538	2.937	3.278
Default φ_s	7.375	3.463	3.835	6.538	2.910	3.254
Enhanced φ_s	**7.113**	3.477	**3.822**	**6.025**	**2.846**	**3.147**
Uniform φ_u	9.938	**3.327**	3.958	6.337	3.227	3.520
Minimal	7.950	3.660	4.067	6.050	3.235	3.500
RandomSy	12.963	4.082	4.938	12.963	4.082	4.938

programs becomes extremely small. Note that the target of *SampleSy* is to exclude all incorrect programs: the lack of incorrect samples may mislead its decision. Since the size of an incorrect program is often large and φ_u is the most concentrated on large programs among all distributions in this experiment, φ_u suffers less on this issue. In String, φ_u sampled less than 5 incorrect programs in only 3% of the turns, while φ_s did the same for 9% of the turns. Please note that this issue does not exist for *EpsSy* since *EpsSy* directly returns when some semantics occurs too many times among samples.

6.6 Exp 3: Comparison of the Sample Size (RQ3)

Procedure. In this experiment, we ran *SampleSy* on all benchmarks with a limitation w on the number of samples, i.e., the sampler only returns at most w samples to the controller in each turn. We ran *SampleSy* three times with $w = 2, 20, 5000$, respectively and measured the number of questions required.

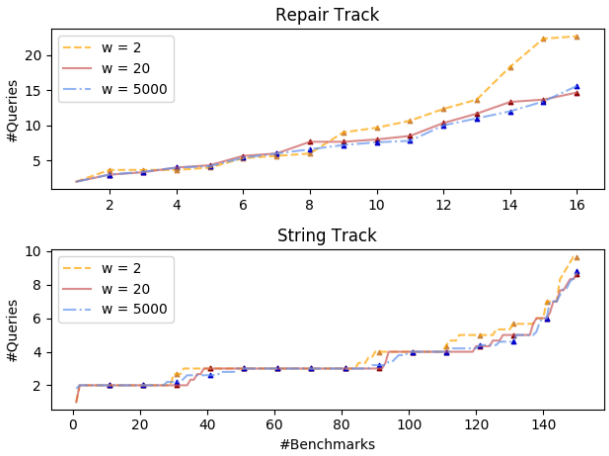


Figure 3. The results of experiment 3. The figure are drawn in the same way as Figure 2.

Results. The results are summarized in Figure 3. For convenience, we use $S(k)$ to represent *SampleSy* with w equal to k. Figure 3 shows that $S(2)$ performs significantly worse than $S(5000)$: On the hardest 30% benchmarks, $S(2)$ takes 50.0% more questions than $S(5000)$ in *Repair* and 12.7% more questions in *String*. However, such a performance gap disappears when w is only increased by 18: the curves representing for $S(20)$ and $S(5000)$ almost coincide in Figure 3. More precisely, $S(20)$ only uses 3.6% more questions than $S(5000)$ in *Repair* and 0.5% more questions in *String*.

This result shows that the performance of *SampleSy* converges rapidly when w increases, which is consistent with Theorem 3.2.

6.7 Exp 4: Comparison of Values of f_ϵ (RQ4)

Procedure. In this experiment, for each $f_\epsilon \in [0, 5]$, we ran *EpsSy* on all benchmarks, and recorded whether the result is correct and the number of questions required.

Results. The results of the experiment are summarized in Figure 4. We make the following observations from it:

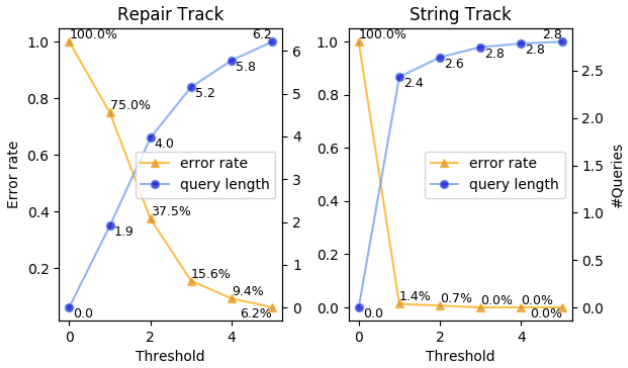


Figure 4. The results of experiment 4 The x-axis represents the value of f_ϵ, the left y-axis represents the error rate while the right y-axis represents the average number of questions required by *EpsSy*.

- For both datasets, the error rate drops exponentially when f_ϵ increases, we are able to obtain a high accuracy with a small f_ϵ. This result is consistent with Theorem 4.6. Moreover, the average number of questions only increases linearly (or even sub-linearly) when f_ϵ increases, which means a much better accuracy can be obtained with only a small loss on the number questions.

- The error rate in *String* drops much faster than that in *Repair* while the number of questions increases much slower in *String*. This is because different termination conditions dominate *EpsSy* in the two datasets: For *Repair*, since the numbers of questions required are generally large, *EpsSy* terminates mainly when the confidence reaches f_ϵ. Therefore the number of questions grows around linearly with f_ϵ while the error rate drops relatively more slowly. For *String*, *EpsSy* terminates mainly when the conditional probability of some semantics is too large. Therefore the number of questions and the error rate is more independent with f_ϵ than *Repair*.

7 Related Work

Question selection in interactive synthesis. As mentioned before, multiple existing interactive synthesis systems [36, 49] already contain a question selection component. In those systems, the question selection components try to select a *distinguishing* question, i.e., at least two programs have different answer. Among all *distinguishing* questions, the selection is random. For example, Mayer et al. [36] use input-output questions and randomly select input within a test suite until a distinguishing input is found. Wang et al. [49] also use input-output questions and randomly mutate existing inputs until a distinguishing input is found. Such a selection component is equivalent to *RandomSy* in our evaluation. Compared to the existing approaches, our approach selects a question within the whole question space that approximates an optimal question, which significantly outperforms *RandomSy* as our evaluation indicates.

Padhi et al. [39] explore another form of question selection problem for input-output questions. Their approach, called *Significant Inputs*, returns a set of questions in each turn and lets the user select which questions to answer among all selected questions. *Significant Inputs* uses a syntactic profile over structured inputs to perform the selection. Compared with *Significant Inputs*, our approach returns one question each turn and does not require a syntactic profile, applicable to unstructured inputs such as integers. Furthermore, our approach is the first to give the theoretical guarantees on disambiguation and the number of required queries. The two approaches can potentially be combined: *Significant Inputs* selects a set of questions while our approaches choose among these questions.

User-driven interactive synthesis. Besides asking easy-to-answer questions, another form of interaction in program synthesis is to demonstrate a set of solutions to the user, and the user decides what actions to take to reduce the ambiguity. The actions include (1) selects a program from the demonstrated programs [36, 52], (2) provides input-output examples to refine the specification [16, 26, 52], and (3) changes some configuration options of the synthesis algorithm [5, 22, 26, 32]. In such interactions there is no question selection problem, as it is the user who decides what action to take. In contrast to these approaches that give an open task to the user, our algorithms shift the burden of question selection from the user to the system, and the user only has to answer deterministic questions.

General framework of interactive program synthesis. Le et al. [30] build an abstract model of interactive program synthesis along three dimensions: *incrementality*, *step-based formulation*, and *feedback-based interaction*. Our algorithms can be viewed as instantiations of the framework, providing feed-back interaction by selecting questions.

Counterexample guided inductive synthesis. Counterexample guided inductive synthesis (CEGIS) [3, 19, 47, 48] assumes the existence of a verification oracle, which could verify whether a synthesized program is correct, and returns a counterexample if it is not. Oracle-guided inductive program synthesis (OGIS) [24] is a generalization of CEGIS, which allows more types of questions to be asked to the oracle. CEGIS and OGIS are similar to interactive program synthesis as humans can be considered as an oracle. However, since OGIS and CEGIS focus on automated oracles, so far there is no practical algorithm for CEGIS/OGIS to minimize the number questions as far as we are aware. Furthermore, many questions considered in CEGIS/OGIS are not easy for human to answer, such as determining whether a program is correct.

Optimization Modulo Theories Solvers. Optimization Modulo Theories (OMT) is an extension of SMT which allows for finding models that optimize given objectives. Since finding the best question according to *minimax branch* is an optimization problem, OMT solvers [6, 7, 27, 33, 44, 45] are related to our work. To deal with an optimization problem, existing OMT solvers require to encode the problem into some pre-designed domain-specific languages. However, encoding *minimax branch*, i.e., Definition 2.7, is a non-trivial challenge: $w\left(\mathbb{P}|_{C\cup\{(q,a)\}}\right)$ is an instance of *weighted model counting*, which is not supported by any existing OMT solver. Therefore, instead of using OMT solvers to find the best question, we designed *SampleSy* and *EpsSy* to approximate *minimax branch* via sampling.

Optimal decision trees. Previously sections have discussed optimal decision trees [28] and optimal question selection are mutually polynomial-time transformable. However, compared with interactive synthesis, the program/question/answer domains in optimal decision tree are usually much smaller, and thus strategies like *minimax branch* can be directly applied. Our work starts from *minimax branch*, and proposes a set of approximation algorithms to make it practical in interactive synthesis.

Active learning. Active learning is a research domain about finding objects with some target properties from a candidate domain by interactively asking questions to some oracle. In

this domain, the question selection problems are also studied to reduce the number of questions or to enhance the efficiency of a fixed number of questions. However, none of them can be directly applied to interactive program synthesis: (i) Some of them focus on specific models and utilizes concrete properties which do not hold in program synthesis, e.g., Bshouty [8] studies exact learning, a sub problem of active learning that focuses on learning an exact model, but for only boolean functions; Schohn and Cohn [43] study specifically for *support vector machines(SVM)*. (ii) Some of the previous works focus on general question selection problems and their results cannot be applied to interactive program synthesis due to the scalability, e.g., Dasgupta [12] analyzes a greedy strategy of which the time complexity is polynomial to the number of candidates, i.e., the size of the program space. (iii) Others only focus on theoretic results, e.g., Bshouty et al. [9] proves multiple theoretic properties of exact learning, but these properties cannot be directly used to design algorithms. As we are aware, this paper is the first work studying the question selection problem for interactive program synthesis.

8 Conclusion

In this paper, we address the question selection in interactive program synthesis with the goal of minimizing the number of questions to the user. Staring from the *minimax branch* strategy, we design two efficient algorithms *SampleSy* and *EpsSy* for two different scenarios: *SampleSy* ensures the result to be correct while *EpsSy* has a bounded error rate. On the theoretical side, we show that both the loss of *SampleSy* comparing with *minimax branch* and the error rate of *EpsSy* are bounded. On the practical side, we implement these two approaches and our evaluation shows that both algorithms outperform the existing strategies significantly.

Appendices

The appendix can be found in the supplementary material [1] and consists of two parts. The first part discusses the relation between optimal question selection and optimal decision tree, and the second part gives the proofs for all other theorems in the paper.

Acknowledgements

We thank Yican Sun and the anonymous PLDI reviewers for valuable feedback on this work. This work is supported in part by the National Key Research and Development Program of China under Grant No. 2017YFB1001803, National Natural Science Foundation of China under Grant Nos. 61922003 and 61672045.

References

[1] [n. d.]. Supplementary Material. https://github.com/jiry17/IntSy.

[2] Micah Adler and Brent Heeringa. 2008. Approximating optimal binary decision trees. In *Approximation, Randomization and Combinatorial Optimization. Algorithms and Techniques*. Springer, 1–9.

[3] Rajeev Alur, Rastislav Bodík, Garvit Juniwal, Milo M. K. Martin, Mukund Raghothaman, Sanjit A. Seshia, Rishabh Singh, Armando Solar-Lezama, Emina Torlak, and Abhishek Udupa. 2013. Syntax-guided synthesis. In *Formal Methods in Computer-Aided Design, FMCAD 2013, Portland, OR, USA, October 20-23, 2013*. 1–8.

[4] Rajeev Alur, Arjun Radhakrishna, and Abhishek Udupa. 2017. Scaling Enumerative Program Synthesis via Divide and Conquer. In *Tools and Algorithms for the Construction and Analysis of Systems - 23rd International Conference, TACAS 2017, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2017, Uppsala, Sweden, April 22-29, 2017, Proceedings, Part I*. 319–336. https://doi.org/10.1007/978-3-662-54577-5_18

[5] Shaon Barman, Rastislav Bodík, Satish Chandra, Emina Torlak, Arka Aloke Bhattacharya, and David Culler. 2015. Toward tool support for interactive synthesis. In *2015 ACM International Symposium on New Ideas, New Paradigms, and Reflections on Programming and Software, Onward! 2015, Pittsburgh, PA, USA, October 25-30, 2015*. 121–136.

[6] Nikolaj Bjørner and Anh-Dung Phan. 2014. νZ - Maximal Satisfaction with Z3. In *6th International Symposium on Symbolic Computation in Software Science, SCSS 2014, Gammarth, La Marsa, Tunisia, December 7-8, 2014*. 1–9. http://www.easychair.org/publications/paper/200953

[7] Nikolaj Bjørner, Anh-Dung Phan, and Lars Fleckenstein. 2015. νZ - An Optimizing SMT Solver. In *Tools and Algorithms for the Construction and Analysis of Systems - 21st International Conference, TACAS 2015, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2015, London, UK, April 11-18, 2015. Proceedings*. 194–199. https://doi.org/10.1007/978-3-662-46681-0_14

[8] Nader H. Bshouty. 1995. Exact Learning Boolean Functions via the Monotone Theory. *Electronic Colloquium on Computational Complexity (ECCC)* 2, 8 (1995). http://eccc.hpi-web.de/eccc-reports/1995/TR95-008/index.html

[9] Nader H. Bshouty, Richard Cleve, Ricard Gavaldà, Sampath Kannan, and Christino Tamon. 1996. Oracles and Queries That Are Sufficient for Exact Learning. *J. Comput. Syst. Sci.* 52, 3 (1996), 421–433. https://doi.org/10.1006/jcss.1996.0032

[10] Venkatesan T Chakaravarthy, Vinayaka Pandit, Sambuddha Roy, Pranjal Awasthi, and Mukesh Mohania. 2007. Decision trees for entity identification: Approximation algorithms and hardness results. In *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*. ACM, 53–62.

[11] Venkatesan T Chakaravarthy, Vinayaka Pandit, Sambuddha Roy, and Yogish Sabharwal. 2009. Approximating decision trees with multiway branches. In *International Colloquium on Automata, Languages, and Programming*. Springer, 210–221.

[12] Sanjoy Dasgupta. 2004. Analysis of a greedy active learning strategy. In *Advances in Neural Information Processing Systems 17 [Neural Information Processing Systems, NIPS 2004, December 13-18, 2004, Vancouver, British Columbia, Canada]*. 337–344. http://papers.nips.cc/paper/2636-analysis-of-a-greedy-active-learning-strategy

[13] Leonardo De Moura and Nikolaj Bjørner. 2008. Z3: An efficient SMT solver. In *International conference on Tools and Algorithms for the Construction and Analysis of Systems*. Springer, 337–340.

[14] Yu Feng, Ruben Martins, Osbert Bastani, and Isil Dillig. 2018. Program synthesis using conflict-driven learning. In *Proceedings of the 39th ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI 2018, Philadelphia, PA, USA, June 18-22, 2018*. 420–435.

[15] Yu Feng, Ruben Martins, Jacob Van Geffen, Isil Dillig, and Swarat Chaudhuri. [n. d.]. Component-based synthesis of table consolidation and transformation tasks from examples. In *Proceedings of the 38th ACM SIGPLAN Conference on Programming Language Design and*

Implementation, PLDI 2017, Barcelona, Spain, June 18-23, 2017.

[16] Joel Galenson, Philip Reames, Rastislav Bodik, Bjorn Hartmann, and Koushik Sen. 2014. CodeHint: dynamic and interactive synthesis of code snippets. In *36th International Conference on Software Engineering, ICSE '14, Hyderabad, India - May 31 - June 07, 2014*. 653–663.

[17] Sumit Gulwani. 2011. Automating string processing in spreadsheets using input-output examples. In *Proceedings of the 38th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL 2011, Austin, TX, USA, January 26-28, 2011*. 317–330.

[18] Sumit Gulwani and Prateek Jain. 2017. Programming by Examples: PL meets ML. In *APLAS*.

[19] Sumit Gulwani, Susmit Jha, Ashish Tiwari, and Ramarathnam Venkatesan. 2011. Synthesis of loop-free programs. In *Proceedings of the 32nd ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI 2011, San Jose, CA, USA, June 4-8, 2011*. 62–73.

[20] Sumit Gulwani, Oleksandr Polozov, and Rishabh Singh. 2017. Program Synthesis. *Foundations and Trends in Programming Languages* 4, 1-2 (2017), 1–119.

[21] Anupam Gupta, Viswanath Nagarajan, and R Ravi. 2017. Approximation algorithms for optimal decision trees and adaptive TSP problems. *Mathematics of Operations Research* 42, 3 (2017), 876–896.

[22] Tihomir Gvero, Viktor Kuncak, and Ruzica Piskac. 2011. Interactive Synthesis of Code Snippets. In *Computer Aided Verification - 23rd International Conference, CAV 2011, Snowbird, UT, USA, July 14-20, 2011. Proceedings*. 418–423.

[23] Susmit Jha, Sumit Gulwani, Sanjit A. Seshia, and Ashish Tiwari. 2010. Oracle-guided component-based program synthesis. In *Proceedings of the 32nd ACM/IEEE International Conference on Software Engineering - Volume 1, ICSE 2010, Cape Town, South Africa, 1-8 May 2010*. 215–224.

[24] Susmit Jha and Sanjit A. Seshia. 2017. A theory of formal synthesis via inductive learning. *Acta Inf.* 54, 7 (2017), 693–726.

[25] Mark Johnson. 1998. PCFG Models of Linguistic Tree Representations. *Computational Linguistics* 24, 4 (1998), 613–632.

[26] Sean Kandel, Andreas Paepcke, Joseph M. Hellerstein, and Jeffrey Heer. 2011. Wrangler: interactive visual specification of data transformation scripts. In *Proceedings of the International Conference on Human Factors in Computing Systems, CHI 2011, Vancouver, BC, Canada, May 7-12, 2011*. 3363–3372.

[27] Daniel Larraz, Albert Oliveras, Enric Rodríguez-Carbonell, and Albert Rubio. 2014. Minimal-Model-Guided Approaches to Solving Polynomial Constraints and Extensions. In *Theory and Applications of Satisfiability Testing - SAT 2014 - 17th International Conference, Held as Part of the Vienna Summer of Logic, VSL 2014, Vienna, Austria, July 14-17, 2014. Proceedings*. 333–350. https://doi.org/10.1007/978-3-319-09284-3_25

[28] Hyafil Laurent and Ronald L Rivest. 1976. Constructing optimal binary decision trees is NP-complete. *Information processing letters* 5, 1 (1976), 15–17.

[29] Vu Le and Sumit Gulwani. 2014. FlashExtract: a framework for data extraction by examples. In *ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI '14, Edinburgh, United Kingdom - June 09 - 11, 2014*. 542–553.

[30] Vu Le, Daniel Perelman, Oleksandr Polozov, Mohammad Raza, Abhishek Udupa, and Sumit Gulwani. 2017. Interactive Program Synthesis. *CoRR* abs/1703.03539 (2017).

[31] Woosuk Lee, Kihong Heo, Rajeev Alur, and Mayur Naik. 2018. Accelerating search-based program synthesis using learned probabilistic models. In *Proceedings of the 39th ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI 2018, Philadelphia, PA, USA, June 18-22, 2018*. 436–449. https://doi.org/10.1145/3192366.3192410

[32] Alan Leung, John Sarracino, and Sorin Lerner. 2015. Interactive parser synthesis by example. In *Proceedings of the 36th ACM SIGPLAN Conference on Programming Language Design and Implementation, Portland, OR, USA, June 15-17, 2015*. 565–574.

[33] Yi Li, Aws Albarghouthi, Zachary Kincaid, Arie Gurfinkel, and Marsha Chechik. 2014. Symbolic optimization with SMT solvers. In *The 41st Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL '14, San Diego, CA, USA, January 20-21, 2014*. 607–618. https://doi.org/10.1145/2535838.2535857

[34] Fan Long and Martin Rinard. 2016. Automatic patch generation by learning correct code. In *Proceedings of the 43rd Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*. 298–312. https://doi.org/10.1145/2837614.2837617

[35] Ruben Martins, Jia Chen, Yanju Chen, Yu Feng, and Isil Dillig. 2019. Trinity: An Extensible Synthesis Framework for Data Science. *PVLDB* 12, 12 (2019), 1914–1917.

[36] Mikaël Mayer, Gustavo Soares, Maxim Grechkin, Vu Le, Mark Marron, Oleksandr Polozov, Rishabh Singh, Benjamin G. Zorn, and Sumit Gulwani. 2015. User Interaction Models for Disambiguation in Programming by Example. In *UIST*. ACM, 291–301.

[37] Sergey Mechtaev, Alberto Griggio, Alessandro Cimatti, and Abhik Roychoudhury. 2018. Symbolic execution with existential second-order constraints. In *Proceedings of the 2018 ACM Joint Meeting on European Software Engineering Conference and Symposium on the Foundations of Software Engineering, ESEC/SIGSOFT FSE 2018, Lake Buena Vista, FL, USA, November 04-09, 2018*. 389–399. https://doi.org/10.1145/3236024.3236049

[38] Hong Mei and Lu Zhang. 2018. Can big data bring a breakthrough for software automation? *Sci. China Inf. Sci.* 61, 5 (2018), 056101:1–056101:3. https://doi.org/10.1007/s11432-017-9355-3

[39] Saswat Padhi, Prateek Jain, Daniel Perelman, Oleksandr Polozov, Sumit Gulwani, and Todd D. Millstein. 2018. FlashProfile: a framework for synthesizing data profiles. *PACMPL* 2, OOPSLA (2018), 150:1–150:28.

[40] Phitchaya Mangpo Phothilimthana, Aditya Thakur, Rastislav Bodík, and Dinakar Dhurjati. 2016. Scaling up Superoptimization. In *Proceedings of the Twenty-First International Conference on Architectural Support for Programming Languages and Operating Systems, ASPLOS '16, Atlanta, GA, USA, April 2-6, 2016*. 297–310. https://doi.org/10.1145/2872362.2872387

[41] Oleksandr Polozov and Sumit Gulwani. 2015. FlashMeta: a framework for inductive program synthesis. In *Proceedings of the 2015 ACM SIGPLAN International Conference on Object-Oriented Programming, Systems, Languages, and Applications, OOPSLA 2015, part of SPLASH 2015, Pittsburgh, PA, USA, October 25-30, 2015*. 107–126.

[42] Zichao Qi, Fan Long, Sara Achour, and Martin Rinard. 2015. An Analysis of Patch Plausibility and Correctness for Generate-and-validate Patch Generation Systems *(ISSTA 2015)*. 24–36.

[43] Greg Schohn and David Cohn. 2000. Less is More: Active Learning with Support Vector Machines. In *Proceedings of the Seventeenth International Conference on Machine Learning (ICML 2000), Stanford University, Stanford, CA, USA, June 29 - July 2, 2000*. 839–846.

[44] Roberto Sebastiani and Patrick Trentin. 2015. OptiMathSAT: A Tool for Optimization Modulo Theories. In *Computer Aided Verification - 27th International Conference, CAV 2015, San Francisco, CA, USA, July 18-24, 2015, Proceedings, Part I*. 447–454. https://doi.org/10.1007/978-3-319-21690-4_27

[45] Roberto Sebastiani and Patrick Trentin. 2015. Pushing the Envelope of Optimization Modulo Theories with Linear-Arithmetic Cost Functions. In *Tools and Algorithms for the Construction and Analysis of Systems - 21st International Conference, TACAS 2015, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2015, London, UK, April 11-18, 2015. Proceedings*. 335–349. https://doi.org/10.1007/978-3-662-46681-0_27

[46] Armando Solar-Lezama, Rodric M. Rabbah, Rastislav Bodík, and Kemal Ebcioglu. 2005. Programming by sketching for bit-streaming programs. In *Proceedings of the ACM SIGPLAN 2005 Conference on Programming Language Design and Implementation, Chicago, IL, USA, June 12-15, 2005*. 281–294. https://doi.org/10.1145/1065010.1065045

[47] Armando Solar-Lezama, Liviu Tancau, Rastislav BodÃŋk, Sanjit A. Seshia, and Vijay A. Saraswat. 2006. Combinatorial sketching for finite programs. *ACM SIGOPS Operating Systems Review* 40, 5 (2006), 404.

[48] Armando Solarlezama. 2008. Program synthesis by sketching. *Dissertations & Theses - Gradworks* (2008).

[49] Chenglong Wang, Alvin Cheung, and Rastislav Bodík. 2017. Interactive Query Synthesis from Input-Output Examples. In *Proceedings of the 2017 ACM International Conference on Management of Data, SIGMOD Conference 2017, Chicago, IL, USA, May 14-19, 2017*. 1631–1634.

[50] Yuepeng Wang, Xinyu Wang, and Isil Dillig. 2018. Relational program synthesis. *PACMPL* 2, OOPSLA (2018), 155:1–155:27.

[51] Yingfei Xiong, Jie Wang, Runfa Yan, Jiachen Zhang, Shi Han, Gang Huang, and Lu Zhang. 2017. Precise Condition Synthesis for Program Repair. In *Proceedings of the 39th International Conference on Software Engineering, ICSE 2017, Buenos Aires, Argentina, May 20-28, 2017*. https://doi.org/10.1109/ICSE.2017.45

[52] Kuat Yessenov, Shubham Tulsiani, Aditya Krishna Menon, Robert C. Miller, Sumit Gulwani, Butler W. Lampson, and Adam Kalai. 2013. A colorful approach to text processing by example. In *The 26th Annual ACM Symposium on User Interface Software and Technology, UIST'13, St. Andrews, United Kingdom, October 8-11, 2013*. 495–504.